Let \( g \) be a finite dimensional restricted Lie algebra over an algebraically closed field \( k \) of prime characteristic \( p \). Then for every simple \( g \)-module \( S \) there exists a linear form \( \chi \) on \( g \), the so-called \( p \)-character of \( S \), such that \( S \) is a simple module for a certain finite dimensional quotient \( U_\chi(g) \) of the universal enveloping algebra \( U(g) \) of \( g \). In case \( g \) is the Lie algebra of a reductive algebraic group \( G \), the linear dual \( g^* \) and \( g \) are \( G \)-isomorphic and so the Jordan decomposition of elements in \( g \) transfers to \( g^* \). Hence \( \chi \) can be decomposed into a sum of its semisimple and nilpotent parts. It is well known that it is enough to consider only simple \( U_\chi(g) \)-modules for nilpotent \( \chi \), but, in general, the structure of these simple modules is still unknown. If \( g = sl_n(k) \), then one can even assume that \( \chi \) is in standard Levi form. In particular \( \chi \) vanishes on the standard Borel subalgebra of \( g \).

The easiest case is when \( \chi \) is regular, that is, when the dimension of the \( G \)-stabilizer of \( \chi \) is equal to the rank of \( G \). In this case every simple \( U_\chi(g) \)-module is isomorphic to a baby Verma module, that is, a module induced from a one-dimensional (restricted) module for the standard Borel subalgebra of \( g \) (if \( \chi \) is in standard Levi form). In several papers Jantzen studied the next simplest case, namely the subregular case, when the dimension of the \( G \)-stabilizer of \( \chi \) is equal to the rank of \( G \) plus two. Motivated by the restricted case \( \chi = 0 \), he introduced a certain category \( C \) of graded \( U_\chi(g) \)-modules whose simple objects (after forgetting the grading) coincide with the simple \( U_\chi(g) \)-modules. In particular, for \( g = sl_n(k) \) and \( n \) not divisible by \( p \), Jantzen [Math. Proc. Camb. Philos. Soc. 126, No. 2, 223-257 (1999; Zbl. 0939.17008)] obtained an explicit description of the simple modules in terms of baby Verma modules and also calculated the extensions between non-isomorphic simple objects in \( C \).

For the case \( g = sl_3(k) \) and \( \chi \) being subregular nilpotent in standard Levi form the author of the paper under review uses Jantzen’s description of the simple modules just mentioned, Jantzen’s filtrations of principal indecomposable modules by baby Verma modules, and Jantzen’s translation principle to work out the structure of the principal indecomposable objects in \( C \). As a consequence, he then also determines the self-extensions of simple objects in \( C \).