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Lusztig’s geometric approach to Hall algebras.

Mathematics Subject Classification 2000: Primary 17B55; 20G99; Secondary 17B50

Keywords: Hall algebra, composition algebra, quiver, nilpotent representation, ℓ-adic sheaf, Frobenius morphism, pure complex of sheaves, Lefschetz fixed point formula, Grothendieck trace formula

Reviewer: Jörg Feldvoss (8086)

The aim of the paper under review is to explain Lusztig’s geometric construction of Hall algebras and compare it with Ringel’s combinatorial construction. The author starts by reformulating the definition of the Hall algebra $\mathcal{H}(q)$ of a finite quiver $Q$ in terms of complex valued functions on the set of isomorphism classes of nilpotent representations of $Q$ over a finite field $\mathbb{F}_q$ with $q$ elements. Here a representation of $Q$ is called nilpotent if every closed directed cycle of $Q$ acts nilpotently on the underlying representation space. Then Lusztig’s geometric approach to define a Hall algebra $\mathcal{K}_q$ in terms of pure complexes of $\ell$-adic sheaves over the algebraic closure of $\mathbb{F}_q$ is described where $\ell$ is a prime number different from the characteristic of $\mathbb{F}_q$. The author also defines a surjective homomorphism from the geometric Hall algebra $\mathcal{K}_q$ to the Hall algebra $\mathcal{H}(q)$ defined earlier. The paper concludes by outlining Lusztig’s geometric construction of the composition algebra which is the subalgebra of $\mathcal{H}(q)$ generated by the simple nilpotent representations of $Q$. 