Leclerc, Bernard; Nazarov, Maxim; Thibon, Jean-Yves
Induced representations of affine Hecke algebras and canonical bases of quantum groups.

Mathematics Subject Classification 2000: 17B37; 20C08; 17B10

Keywords: Affine Hecke algebra, induction product, irreducible representation, Grothendieck group, Hopf algebra, quantum group, canonical basis, quantum flag minor, evaluation module, irreducibility criterion

Reviewer: Jörg Feldvoss (8086)

Let $GL_m$ be the general linear group over a non-Archimedean local field and let $C_m$ denote the category of certain smooth representations of finite length for $GL_m$. Then $C_m$ is equivalent to the category $C(\hat{H}_m)$ of finite-dimensional complex representations of the affine Hecke algebra $\hat{H}_m$ of $GL_m$. Moreover, one has natural induction functors $(M_1, M_2) \mapsto M_1 \otimes M_2$ from $C_m \times C_m$ to $C_m + C_m$ and from $C(\hat{H}_m) \times C(\hat{H}_m)$ to $C(\hat{H}_m + \hat{H}_m)$ which correspond to each other via the equivalence of categories mentioned above. In the paper under review the authors study under which conditions the induction product $L_1 \otimes L_2$ of irreducible objects $L_1$ and $L_2$ of these categories is irreducible.

Let $R_m$ denote the complexified Grothendieck group of $C_m$ or $C(\hat{H}_m)$ for a positive integer $m$ and set $R_0 := C$. It was shown by I. N. Bernstein and A. V. Zelevinsky [Ann. Sci. Éc. Norm. Supér., IV. Sér. 10, 441-472 (1977; Zbl 0412.22015)] that the induction and restriction functors make $R = \bigoplus_{m \geq 0} R_m$ into a Hopf algebra. According to a result of A. V. Zelevinsky [Ann. Sci. Éc. Norm. Supér., IV. Sér. 13, 165-210 (1980; Zbl 0441.22014)], $R$ is a tensor product of isomorphic copies of a Hopf algebra $R$ which is isomorphic to the algebra of polynomial functions on the group of upper triangular unipotent $\mathbb{Z} \times \mathbb{Z}$-matrices with only finitely many non-zero entries off the main diagonal. As such $R$ has a quantum deformation $R_v$ endowed with a (dual) canonical basis $B_v$ which specializes for $v \to 1$ to a canonical basis $B$ of $R$. The canonical basis $B$ corresponds to the basis of $R$ consisting of the isomorphism classes of irreducibles representations and multiplying elements of $B$ is the same as taking induction products of irreducibles representations in $R$.

A distinguished subset of $B_v$ consists of quantum flag minors. Note that quantum flag minors of degree $m$ correspond to the so-called evaluation modules for $\hat{H}_m$ which are obtained by lifting the irreducible modules of the finite-dimensional Hecke algebra $H_m$. The multiplicative properties of $B_v$ have been studied by Arkady Berenstein and Andrei Zelevinsky [Gelfand, Sergej (ed.) et al., I. M. Gelfand seminar. Part 1: Papers of the Gelfand seminar in functional analysis held at Moscow University, Russia, September 1993. Providence, RI: American Mathematical Society. Adv. Sov. Math. 16(1), 51-89 (1993; Zbl 0794.17007)] who conjectured that the product of two elements of $B_v$ belongs to $B_v$ up to a power of $v$ if and only if these elements commute up to a power of $v$. The authors prove this conjecture in the special case of quantum flag minors for which there is a combinatorial criterion of $v$-commutativity due to the first author and Andrei Zelevinsky [Olshanski, G. I. (ed.), Kirillov’s seminar on representation theory. Providence, RI: American Mathematical Society. Transl., Ser. 2, Am. Math. Soc. 181(35), 85-108 (1998; Zbl 0894.14021)]. As a consequence the authors obtain an explicit irreducibility criterion for the induction product of two evaluation modules. Finally, this irreducibility criterion is extended to induction products of any finite number of evaluation modules by exploiting the representation theory of the affine quantum group $U_v(\mathfrak{sl}_N)$. 