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Artin–Hasse exponentials of derivations.

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It is well-known that for a nilpotent derivation $D$ of a (not necessarily associative) algebra $A$ over a field of characteristic zero the finite sum \( \exp(D) = \sum_{r=0}^{\infty} D^r / r! \) is an automorphism of $A$. Over ground fields of prime characteristic $p$ only the first $p$ terms of $\exp(D)$ make sense but even if $D^p = 0$, then the truncated exponential $E(D) = \sum_{r=0}^{p-1} D^r / r!$ is not always an automorphism of $A$. Nevertheless, as any algebra automorphism the truncated exponential $E(D)$ of a derivation $D$ of $A$ satisfying $D^p = 0$ can be employed to obtain a new grading of $A$ from a given one. The paper under review elaborates on this observation by showing that Artin-Hasse exponentials of nilpotent graded derivations of (not necessarily associative) graded algebras over fields of prime characteristic can be used to produce new cyclic gradings from a given one. The author also discusses the connection of grading switching via Artin-Hasse exponentials and toral switching for restricted Lie algebras. Contrary to toral switching, Artin-Hasse exponentials can be employed to obtain gradings over cyclic $p$-groups of order larger than $p$. As an application the author constructs certain cyclic gradings of Zassenhaus Lie algebras for $p > 3$. Moreover, Marina Avitabile and the author [J. Algebra 293, No. 1, 34-64 (2005; Zbl. pre02235767)] have used the Artin-Hasse exponential of a nilpotent graded derivation to produce certain cyclic gradings of Block Lie algebras.