Let $v$ be an indeterminate and let $U$ be the quantized enveloping algebra over $\mathbb{Q}(v)$ associated to a symmetrizable Cartan matrix. Any choice of a reduced expression for the longest element $w_0$ in the Weyl group induces a total ordering on the positive roots. Using Lusztig’s braid group action on $U$ one can then define divided power root vectors $E^{(a)}_\alpha$ for every positive root $\alpha$ and every non-negative integer $a$.

The main result of the paper under review is an explicit commutation formula for two divided power root vectors which generalizes Lemma 1.7 of C. de Concini and V. G. Kac [in: Operator algebras, unitary representations, enveloping algebras, and invariant theory, Paris, 1989, Progress in Mathematics, Vol. 92, Birkhäuser, Boston, pp. 471-506 (1990; Zbl. 0738.17008)]. As a consequence the author shows that the integral form of the positive part $U^+$ of $U$ over $\mathbb{Z}[v, v^{-1}]$ has a PBW-type basis in terms of the $E^{(a)}_\alpha$, where the ordering of the positive roots in the monomials is arbitrary (unlike the Dyer-Lusztig basis where the ordering depends on the choice of the reduced expression for $w_0$). Moreover, a variation of the commutation formula is used to prove that certain PBW-like bases are orthogonal for Kashiwara’s bilinear form on $U^+$. A similar result for the bilinear form obtained from the Drinfeld dual was established before by S. Z. Levendorskij and Ya. S. Sojbel’man [J. Geom. Phys. 7, No. 2, 241-254 (1990; Zbl. 0729.17009)] as well as G. Lusztig [Introduction to Quantum Groups, Progress in Mathematics, Vol. 110, Birkhäuser, Boston/Basel/Berlin (1993; Zbl. 0788.17010)].