Let $\mathfrak{g}$ be a split finite dimensional semisimple Lie algebra over a field of characteristic zero. Let $q$ be an element in the ground field of $\mathfrak{g}$ that is transcendental and let $\mathcal{U}_q(\mathfrak{g})$ denote the quantized universal enveloping algebra of $\mathfrak{g}$ at $q$ with standard generators $X^\pm_1, \ldots, X^\pm_r$ and $K^\pm_1, \ldots, K^\pm_r$. De Concini, Kac, and Procesi [Some quantum analogues of solvable Lie groups, in: Geometry and analysis. Papers presented at the Bombay colloquium, India, January 6–14, 1992. Oxford: Oxford University Press. Stud. Math., Tata Inst. Fundam. Res. 13, 41-65 (1995; Zbl. 0878.17014)] defined for a given element $w$ of the Weyl group of $\mathfrak{g}$ certain subalgebras $\mathcal{U}_w$ of $\mathcal{U}_q(\mathfrak{g})$ generated by the Lusztig root vectors obtained from reduced expressions of $w$ and showed that these subalgebras are independent of the choice of the reduced expression.

The goal of the paper under review is to study the prime ideals of $\mathcal{U}_w$ invariant under the conjugation action of the group-like elements $H := \langle K^\pm_1, \ldots, K^\pm_r \rangle$ of $\mathcal{U}_q(\mathfrak{g})$. The main results are as follows: 1) an explicit description of these invariant prime ideals using Demazure modules, 2) a construction of a small generating set for each invariant prime ideal, and 3) an identification of the poset structure on invariant prime ideals with a Bruhat interval. Even in the special case of quantum matrices 1) and 2) are new. The proof uses Gorelik’s investigation of the spectra of quantum Bruhat cell translates [J. Algebra 227, No. 1, 211-253 (2000; Zbl. 1038.17006)], Joseph’s results on generating sets for ideals of the quantized coordinate ring of the simply connected semisimple algebraic group $G$ with Lie algebra $\mathfrak{g}$ [C. R. Acad. Sci. Paris Sér. I Math. 321, No. 2, 135140 (1995)], and an interpretation of $\mathcal{U}_w$ as a quantized function algebra on the Schubert cell $B_+ w \cdot B_+$. Similar results are also obtained for vanishing ideals of torus orbit closures of symplectic leaves of related Poisson structures on Schubert cells in flag varieties. The results of the paper under review play a crucial role in the author’s recent classification of the $H$-invariant prime ideals of arbitrary quantum partial flag varieties of $G$ [Proc. Am. Math. Soc. 138, No. 4, 1249-1261 (2010; Zbl. pre05692519)].