Let $G$ be a connected simply connected split semisimple algebraic group of rank $r$ over a field $\mathbb{K}$ of characteristic zero and let $B$ denote the standard Borel subgroup of $G$ corresponding to the positive roots. Moreover, let $\mathfrak{g}$ denote the Lie algebra of $G$, let $q$ be an element in $\mathbb{K}$ that is transcendental over $\mathbb{Q}$ and let $U_q(\mathfrak{g})$ denote the quantized universal enveloping algebra of $\mathfrak{g}$ at $q$ with standard generators $X_{\pm 1}^\pm, \ldots, X_{\pm r}^\pm$ and $K_{\pm 1}^\pm, \ldots, K_{\pm r}^\pm$. For $I \subseteq \{1, \ldots, r\}$ denote by $P_I \supseteq B$ the standard parabolic subgroup of $G$ corresponding to $I$. Finally, let $R_q[G/P_I]$ denote the quantized coordinate ring of the partial flag variety $G/P_I$. The ring $R_q[G/P_I]$ is a deformation of the coordinate ring of the multicone over $G/P_I$ and is invariant under the conjugation action of the group-like elements $H := \langle K_{\pm 1}^\pm, \ldots, K_{\pm r}^\pm \rangle$ of $U_q(\mathfrak{g})$.

The goal of the paper under review is to study the $H$-invariant prime ideals of $R_q[G/P_I]$ that do not contain the augmentation ideal. The main result is a bijection between these invariant prime ideals and certain pairs of Weyl group elements. In particular, all these invariant prime ideals are completely prime. Moreover, the author conjectures that the inclusion of invariant prime ideals is reflected in the Bruhat order of the components of the parametrizing pairs. The proof of the main result uses techniques of Joseph [Quantum groups and their primitive ideals. Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. 29. Berlin: Springer-Verlag. ix, 383 p. (1995; Zbl. 0808.17004)] similar to those used in establishing a natural bijection between primitive ideals in $R_q[\text{SL}(n)]$ and symplectic leaves of the associated Poisson group $\text{SL}(n, \mathbb{C})$ due to Hodges and Levasseur [J. Algebra 168, No. 2, 455-468 (1994; Zbl. 0814.17012)] as well as in Gorelik’s investigation of the spectra of quantum Bruhat cell translates [J. Algebra 227, No. 1, 211-253 (2000; Zbl. 1038.17006)]. As a consequence one obtains a stratification of the $H$-invariant prime spectrum of $R_q[G/P_I]$ and then the strata are related to $H$-invariant prime ideals of the algebras investigated by the author in a previous paper [Proc. Lond. Math. Soc. (3) 101, No. 2, 454-476 (2010; Zbl. 1229.17020)]. Analogous results are also obtained for quantum deformations of the coordinate rings of the cones $\text{Spec} \left( \bigoplus_{n \in \mathbb{Z}_{\geq 0}} H^0(G/P_I, \mathcal{L}_n) \right)$ over $G/P_I$ for certain dominant weights.