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The classical Hom-Yang-Baxter equation and Hom-Lie bialgebras.

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In the first part of the paper the analogue of the classical Yang-Baxter equation, the classical Hom-Yang-Baxter equation, is defined, and then it is shown how to construct from a solution of the classical Yang-Baxter equation for a Lie algebra $L$ over a field $k$ of characteristic zero and a Lie algebra endomorphism $\alpha : L \to L$ solutions of the classical Hom-Yang-Baxter equation for the associated multiplicative Hom-Lie algebra $(L,\alpha \circ [-,-],\alpha)$. This construction is illustrated for the three-dimensional split complex simple Lie algebra. Here a Hom-Lie algebra $(L,[-,-],\alpha)$ consists of a vector space $L$ over $k$, an antisymmetric $k$-linear map $[-,-] : L \otimes L \to L$, and a $k$-linear endomorphism $\alpha : L \to L$ such that the Hom-Jacobi identity 

$$[[x,y],\alpha(z)] + [[z,x],\alpha(y)] + [[y,z],\alpha(x)] = 0$$

holds for all $x,y,z \in L$. The Hom-Lie algebra $(L,[-,-],\alpha)$ is called multiplicative if $\alpha([x,y]) = [\alpha(x),\alpha(y)]$ for all $x,y \in L$. In this generality this notion was introduced for the first time in [J. Algebra 295, No. 2, 314-361 (2006; Zbl. 1138.17012)] by Hartwig, Larsson, and Silvestrov.

The dual notion of a (co-multiplicative) Hom-Lie coalgebra has been defined by Makhlouf and Silvestrov in [J. Algebra Appl. 9, No. 4, 553-589 (2010; Zbl. 1259.16041)]. In the paper under review the author defines a (multiplicative) Hom-Lie bialgebra consisting of a (multiplicative) Hom-Lie algebra structure and a (co-multiplicative) Hom-Lie coalgebra structure together with a compatibility condition which is a 1-cocycle condition in the Hom-Lie algebra cohomology introduced by Makhlouf and Silvestrov in [Forum Math. 22, No. 4, 715-739 (2010; Zbl. 1201.17012)]. The author also defines a (multiplicative) coboundary Hom-Lie algebra as a (multiplicative) Hom-Lie algebra for which the cobracket is a 1-coboundary in Hom-Lie algebra cohomology. Then a coboundary Hom-Lie algebra is called quasi-triangular if its 1-coboundary is a solution of the classical Hom-Yang-Baxter equation. In the special case $\alpha = \text{id}_L$ one obtains exactly a Lie bialgebra (resp. a coboundary Lie bialgebra resp. a quasi-triangular Lie bialgebra) as defined by Drinfeld. Among other things sufficient conditions for a multiplicative Hom-Lie algebra being a multiplicative coboundary Hom-Lie
algebra and necessary as well as sufficient conditions for a coboundary Hom-Lie algebra being a quasi-triangular Hom-Lie bialgebra are given. Similarly to the first part of the paper, the author shows how to construct Hom-Lie bialgebras (resp. coboundary Hom-Lie bialgebras resp. quasi-triangular Hom-Lie bialgebras) from Lie bialgebras (resp. coboundary Lie bialgebras resp. quasi-triangular Lie bialgebras). In the last section of the paper, it is discussed when a perturbation of the cobracket of a multiplicative Hom-Lie bialgebra is again a multiplicative Hom-Lie bialgebra with the same bracket $[-, -]$ and the same twist $\alpha$. 