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Irreducible representations of the Hamiltonian algebra $H(2r; n)$.  

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Reviewer: Jörg Feldvoss (8086)

Let $n = (n_1, \ldots, n_{2r})$ be an arbitrary $2r$-tuple of positive integers and let $L = H(2r; n)$ denote a graded Hamiltonian Lie algebra over an algebraically closed field of characteristic $p > 2$. The goal of the paper under review is to use the setup of generalized restricted Lie algebras to prove that, up to finitely many exceptions, irreducible representations of $L$ with generalized $p$-character of height at most $\min \{ p^{n_i} - p^{n_i-1} \mid 1 \leq i \leq 2r \} - 2$ are induced from an irreducible representation of the distinguished maximal subalgebra $L_0$ of $L$. The main technical tool is an extension of the definition of a $C$-category to graded Hamiltonian Lie algebras. The concept of a $C$-category involves an $L$-module structure, its restriction to $L_0$, a module structure coming from the defining divided power algebra of $L$, and several compatibility conditions, and was introduced by Skryabin [Independent systems of derivations and Lie algebra representations, in: Algebra and Analysis. Proceedings of the International Centennial Chebotarev Conference, Kazan, Russia, June 5-11, 1994, Arslanov, M. M. (ed.) et al. Berlin: Walter de Gruyter. 115-150 (1996; Zbl. 0878.17004)] to study representations of generalized Jacobson-Witt algebras. In particular, for $\chi$ of height 0 (and $p > r$) or $\chi$ of height 1 the authors obtain the number of isomorphism classes of simple modules with a generalized $p$-character $\chi$ and their dimensions. This generalizes the results of Holmes and Zhang [J. Pure Appl. Algebra 173, No. 2, 135-165 (2002; Zbl. 1007.17014)] and Zhang [Commun. Algebra 30, No. 11, 5393-5429 (2002; Zbl. 1015.17018)] for the restricted case $n = (1, \ldots, 1)$. It would be interesting to compare the author’s results for the smallest Hamiltonian Lie algebra $H(2, (1, 1))$ with those obtained by Koreshkov [Izv. Vyssh. Uchebn. Zaved. Mat. 1978, No. 10(197), 37-46 (1978; Zbl. 0403.17006)] who gave a complete description of all isomorphism classes of the simple modules in this case.