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Nilpotent orbits in the Witt algebra $W_1$.

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Two decades ago Premet [Mat. Sb. 182, No. 5, 746-773 (1991; Zbl. 0737.17006)] investigated the geometry of the nilpotent cone of the Jacobson-Witt algebras $W_n$ over an algebraically closed field $\mathbb{F}$ of prime characteristic $p \geq 3$. He proves that the nilpotent cone $\mathcal{N}$ of $W_n$ is an irreducible complete intersection of codimension $n$ in $W_n$. Moreover, let $\text{Aut}(W_n)$ denote the automorphism group of $W_n$ and set $D := \partial_1 + x_1^{p-1}\partial_2 + \cdots + x_1^{p-1}\cdots x_{n-1}^{p-1}\partial_n$, where $\partial_1, \partial_2, \ldots, \partial_n$ denote the partial derivatives of the truncated polynomial algebra $\mathbb{F}[x_1, \ldots, x_n]/(x_1^p, \ldots, x_n^p)$. Then the nilpotent orbit $\text{Aut}(W_n)D$ is open in $\mathcal{N}$ and its complement $\mathcal{N} \setminus \text{Aut}(W_n)D$ consists exactly of the singular points of $\mathcal{N}$.

In the paper under review the authors decompose the nilpotent cone of the Witt algebra for $p > 3$ into distinct $\text{Aut}(W_1)$-orbits, determine the dimensions of the closures of these orbits, and characterize the partial order of the nilpotent orbits induced by inclusion of their closures. Contrary to the Lie algebra of a reductive algebraic group, where there are only finitely many nilpotent orbits, the Witt algebra has infinitely many nilpotent orbits. At the end of the paper the authors show that any Jacobson-Witt algebra $W_n$ has infinitely many nilpotent orbits, and they conjecture that the same holds for the other restricted simple Lie algebras of Cartan type.