Let $\mathfrak{g}$ be the Lie algebra of a reductive algebraic group $G$ that is defined over an algebraically closed field $F$ of prime characteristic $p$. The main result of the paper under review is a sufficient condition for the simplicity of a $\mathfrak{g}$-module that is induced from a simple module over a parabolic subalgebra $\mathfrak{p}_I$ of $\mathfrak{g}$. Such a $\mathfrak{g}$-module is simple provided a certain polynomial function on the linear dual of a common Cartan subalgebra of $\mathfrak{g}$ and $\mathfrak{p}_I$ does not vanish on the highest weight of the simple $\mathfrak{p}_I$-module.

In a previous paper the author had to assume that the $p$-character of the induced module vanishes on the unipotent radical of the opposite parabolic subalgebra of $\mathfrak{p}_I$. Under this hypothesis the condition on the highest weight is also necessary. In the present paper it is shown that the sufficiency of the condition remains valid without this restriction on the $p$-character. So the main result of the paper under review answers one half of a question of Friedlander and Parshall [Am. J. Math. 112, No. 3, 375-395 (1990; Zbl.0714.17007)] under some mild assumptions on $G$, $\mathfrak{g}$, and $p$.

In a preprint [A criterion for (the) irreducibility of parabolic baby Verma modules of reductive Lie algebras, arXiv.1404.4945v2, 2014] Li, Shu, and Yao gave another sufficient condition for the simplicity of a $\mathfrak{g}$-module induced from a simple module over a parabolic subalgebra for $\mathfrak{g} = \mathfrak{sl}_n(F)$, $\mathfrak{so}_n(F)$, or $\mathfrak{sp}_{2n}(F)$ when the $p$-character is of standard Levi form. At the end of his paper the author shows by an example that his simplicity criterion generalizes the one given by Li, Shu, and Yao.