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Error Rate of $S + N$ Selection Combining M-ary NCFSK in Nakagami Fading

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***Abstract*— The performance of M-ary orthogonal noncoherent frequency-shift keying (NCFSK) with N branch signal-*plus*-noise ($S+N$) selection combining (SC) in Nakagami-m fading is studied. Both independent, identically distributed and independent, nonidentically distributed diversity branches are considered. Two $S+N$ SC receiver structures are studied and their performances are analyzed. Tractable expressions are derived for the symbol error rate performance of these receiver structures in Nakagami-m fading with integer values of m. The performances of the $S+N$ SC combiners are compared to those of classical SC and square-law combining (SLC). The effects of the modulation order, the fading parameter, the average decay factor and the number of diversity branches on the performance of $S+N$ SC are examined and compared to the cases of classical SC and SLC. Monte Carlo simulation results are presented to test the validity of our analytical expressions.**

I. Introduction

Diversity is used in wireless channels to combat the effects of multipath fading. Many diversity combining schemes such as maximal ratio combining (MRC), equal-gain combining (EGC) and selection combining (SC) have been well studied in the literature. Fewer results are available on the performance of signal-*plus*-noise ($S+N$) combining. Unlike classical SC combining where the branch having the largest signal-to-noise ratio (SNR) is selected for data recovery, in $S+N$ SC combining the diversity branch having the largest square-law detector output is chosen for detection. In [1] Chyi *et al* analyzed the performance of M-ary noncoherent frequency-shift keying (NCFSK) with $S+N$ SC combining in independent and identically distributed (i.i.d) Rayleigh fading channels. Annavajjala *et al* [2] proposed and analyzed a generalized selection combining scheme receiver for M-ary NCFSK on i.i.d Rayleigh fading channels where, for each of the M hypotheses, the receiver combines L largest outputs among N available square-law outputs before the detection process. In was shown that the scheme in [2] encompasses schemes previously reported in [1] and [3] for $L = 1$ and $L = N$, respectively. In [4], two new $S+N$ combining receivers were introduced and their performances were analyzed in i.i.d Rayleigh fading. A comparison of various selection combining schemes for the limited case of binary orthogonal NCFSK was presented in [5] for Rayleigh fading. The symbol error rate of M-ary orthogonal NCFSK with $S+N$ SC in Rician fading was obtained in [6] and the performance of binary orthogonal NCFSK with only two diversity branches in Nakagami-m fading was analyzed in [7].

Although the performance of binary and M-ary orthogonal NCFSK with $S+N$ SC in Rayleigh fading is well studied in the literature, to the best of the authors' knowledge, there are no reported studies on M-ary NCFSK with $S+N$ SC in Nakagami-m fading channels. In this paper, we obtain tractable analytical expressions for the performance of M-ary orthogonal NCFSK for two $S+N$ SC receiver designs in Nakagami-m fading with N branch diversity. The two $S+N$ receiver designs to be investigated in this paper are referred to as Model 1 and Model 2 in the sequel. We assume the fadings on the branches are independent but not identically distributed (i.n.d) and do not necessarily have the same fading parameter. We also derive analytical expressions for the performance of M-ary orthogonal NCFSK with classical SC in i.n.d Nakagami fading. These results are new. Numerical examples are given to compare the performance of $S+N$ SC with those of classical SC and square-law combining in Nakagami-m fading.

The remainder of this paper is organized as follows. In Section II, we obtain tractable expressions for the symbol error rate (SER) of M-ary orthogonal NCFSK for $S+N$ SC Model 1 in Nakagami-m fading. In Section III, bit error rate (BER) expressions are derived for the performance of binary orthogonal NCFSK with $S+N$ SC Model 2 in Nakagami-m fading. In Section IV, we study the performance of M-ary orthogonal NCFSK with classical SC in i.n.d Nakagami-m fading. In Section V, we present several numerical examples and we test the validity of our analytical results using Monte Carlo results. We also compare the performances of $S+N$ SC Model 1 and Model 2 with those of classical SC and SLC. Finally, some conclusions are drawn in Section VI.

II. SER of $S+N$ SC Model 1 in Nakagami-m Fading

In this section, we derive the SER of $S+N$ SC Model 1 with N-branch M-ary orthogonal NCFSK signaling in Nakagami-m fading. We assume that the fading process is slow and frequency nonselective and the diversity branches are independent but not necessarily identically distributed. We also assume that the fading parameter m is integer. Perfect knowledge of symbol timing is assumed at the receiver and for each diversity branch, we implement an optimum receiver in the format of a matched filter followed by a square-law detector (SLD).

Let $X_{im}, i = 1, \cdots, N, , m = 1, \cdots, M$ denote the output of the SLD for the mth symbol on the ith diversity branch. Assuming the first symbol in the alphabet is transmitted, we write the output of the SLDs as

$$X_{i1} = |2E_s\alpha_i e^{j\theta_i} + N_{i1}|^2 \quad (1)$$

$$X_{im} = |N_{im}|^2, \quad i = 1, \cdots, N, m = 2, \cdots, M \quad (2)$$

where $N_{im}, i = 1, \cdots, N, m = 1, \cdots, M$ are i.i.d zero-mean complex Gaussian random variables with variance $4E_sN_0$ and

$\alpha_i e^{j\theta_i}, i = 1, \cdots, N$ are the complex channel gains. The fading is assumed to be i.n.d.

We assume that the first symbol in the alphabet is transmitted. Then, an error occurs if $\max_i\{X_{i1}\} < \max_{\substack{i,m \\ m\neq 1}} \{X_{im}\}$. One can show that the SER can be written as [6]

$$P_s = \Pr\left[\max_i\{X_{i1}\} < \max_{\substack{i,m\\ m\neq 1}}\{X_{im}\}\right] \quad (3a)$$

$$= N(M-1) \times \int_0^\infty \left(\prod_{i=1}^N F_{X_{i1}}(x)\right)\left(F_{X_{22}}(x)\right)^{NM-N-1} f_{X_{22}}(x)dx \quad (3b)$$

where $F_X(\cdot)$ and $f_X(\cdot)$ represent the cumulative distribution function (CDF) and probability density function (PDF) of the RV X, respectively. Note that to obtain (3b) from (3a), we have used the fact that the random variables (RVs) $X_{im}, i = 1, \cdots, N, m = 2, \cdots, M$ are i.i.d. The BER can be computed from the SER using [8]

$$P_b = \frac{M}{2(M-1)}P_s. \quad (4)$$

To continue with our analysis, we first obtain the PDF and CDF of X_{i2}. From (2), one notes that X_{i2} is a central chi-squared RV with two degrees of freedom. Hence, its PDF and CDF is given by

$$f_{X_{i2}}(x) = \frac{1}{4E_sN_0}\exp\left(-\frac{x}{4E_sN_0}\right) \quad (5)$$

and

$$F_{X_{i2}}(x) = 1 - \exp\left(-\frac{x}{4E_sN_0}\right) \quad (6)$$

respectively.

To obtain the CDF of $X_{i1}, i = 1, \cdots, N$, we first write it as

$$\begin{aligned} X_{i1} &= |2E_s\alpha_i e^{j\theta_i} + N_{i1}|^2 \\ &= |2E_s\alpha_i + e^{-j\theta_1}N_{i1}|^2|e^{j\theta_i}|^2 \\ &= |2E_s\alpha_i + N_i|^2 \\ &= |2E_s\alpha_i + N_i^I|^2 + |N_i^Q|^2 \end{aligned} \quad (7)$$

where $N_i = e^{-j\theta_i}N_{i1}$ and the fixed phase term $e^{-j\theta_i}$ is absorbed into the noise term N_{i1} without changing its statistics [8, p. 292] to yield N_1. Under the static channel condition, X_{i1} is a non-central chi-squared RV with two degrees of freedom and its CDF is given by

$$F_{X_{i1}|\alpha_i,\theta_i}(x) = 1 - Q_1\left(\frac{2E_s\alpha_i}{\sigma}, \frac{\sqrt{w}}{\sigma}\right) \quad (8)$$

where $\sigma = \sqrt{2E_sN_0}$ and where $Q_1(a, b)$ is the first order Marcum Q-function [9, eqn. (4.11)].

To obtain the unconditional CDF of X_{i1}, we average $F_{X_{i1}|\alpha_i,\theta_i}(x)$ over the joint pdf of (α_i, θ_i) to get

$$\begin{aligned} F_{X_{i1}}(x) &= \int_{\alpha_i=0}^{\infty}\int_{\theta_i=0}^{2\pi} F_{X_{i1}|\alpha_i,\theta_i}(x) f(\alpha_i,\theta_i)d\alpha_i d\theta_i \\ &= \int_{\alpha_i=0}^{\infty}\left(1 - Q_1\left(\frac{2E_s\alpha_i}{\sigma}, \frac{\sqrt{x}}{\sigma}\right)\right) \\ &\times \left\{\int_{\theta_i=0}^{2\pi} f_{\alpha_i,\theta_i}(\alpha_i,\theta_i)d\theta_i\right\}d\alpha_i \quad (9a) \\ &= 1 - \int_{\alpha_i=0}^{\infty} Q_1\left(\frac{2E_s\alpha_i}{\sigma}, \frac{\sqrt{x}}{\sigma}\right) f_{\alpha_i}(\alpha_i)d\alpha_i. \quad (9b) \end{aligned}$$

In Nakagami-m fading, the marginal PDF of α_i is given by [9, eqn. (2.20)]

$$p_{\alpha_i}(\alpha_i) = \frac{2\alpha_i^{2m_i-1}}{\Gamma(m_i)}\left(\frac{m_i}{\Omega_i}\right)^{m_i}\exp\left(-\frac{m_i\alpha_i^2}{\Omega_i}\right) \quad (10)$$

where m_i is the fading parameter corresponding to the ith channel, $\Omega_i = E(\alpha_i^2)$ and $E(\cdot)$ denotes the expectation operation.

Substituting (10) in (9b) and using [10, eq. (9)], and after some mathematical simplifications, we obtain the unconditional CDF of X_{i1} as

$$F_{X_{i1}}(x) = 1 - \exp\left(-\frac{x}{2\sigma^2}\frac{m_i}{m_i+\bar{\gamma}_i}\right)\sum_{k=0}^{m_i-1}\sum_{l=0}^{k}\Theta_{k,l,i}\left(\frac{x}{2\sigma^2}\right)^l \quad (11a)$$

where $\bar{\gamma}_i = \Omega_iE_s/N_0$ is the average SNR on the ith branch and

$$\Theta_{k,l}(m_i,\bar{\gamma}_i) = \frac{\epsilon_k(m_i,\bar{\gamma}_i)\binom{k}{k-l}\bar{\gamma}_i^{\,l+1}m_i^k}{l!\,(m_i+\bar{\gamma}_i)^{l+k+1}} \quad (11b)$$

$$\epsilon_k(m_i,\bar{\gamma}_i) = \begin{cases} 1, & k_i < m_i - 1 \\ \dfrac{m_i+\bar{\gamma}_i}{\bar{\gamma}_i}, & k_i = m_i - 1 \end{cases}. \quad (11c)$$

The SER of M-ary NCFSK with $S+N$ SC Model 1 in Nakagami-m fading can now be computed by substituting (5), (6) and (11) in (3). The result is

$$P_s = N(M-1)\int_0^\infty \prod_{i=1}^N\left(1 - \exp\left(-x - \frac{x}{2\sigma^2}\frac{m_i}{m_i+\bar{\gamma}_i}\right) \times \sum_{k=0}^{m_i-1}\sum_{l=0}^{k}\Theta_{k,l}(m_i,\bar{\gamma}_i)\left(\frac{x}{2\sigma^2}\right)^l\right)\left(1-\exp(-x)\right)^N dx. \quad (12)$$

For i.n.d Rayleigh fading ($m_i = 1, i = 1, \cdots, N$), one can show that (12) reduces to [6, eq. (15)], as expected.

For i.i.d Nakagami-m fading, after some mathematical manipulation and integral evaluations, we obtain a closed-form expression as

$$P_s = N(M-1)\sum_{v=0}^{NM-N-1}\sum_{n=0}^{N}(-1)^{v+n}\binom{MN-N-1}{v} \times \binom{N}{n}\exp\left(-\left(\frac{nm}{m+\bar{\gamma}}+1+v\right)\right)\sum_{\underline{k}^N=\underline{0}^N}^{\underline{m-1}^N} \times \sum_{\underline{l}^N=\underline{0}^N}^{\underline{k}^N}\left\{\prod_{p=1}^{n}\Theta_{k,l}(m_p,\bar{\gamma}_p)\right\}\frac{\left(\sum_{p=1}^{n} l_p\right)!}{\left(\frac{nm}{m+\bar{\gamma}}+1+v\right)^{1+\sum_{p=1}^{n} l_p}} \tag{13}$$

where $\underline{C}^N$ is a constant vector of dimension N with each element being C and we define the summation $\sum_{\underline{\kappa}^N=\underline{\eta}^N}^{\underline{\nu}^N}$ as

$$\sum_{\underline{\kappa}^N=\underline{\eta}^N}^{\underline{\nu}^N} \triangleq \sum_{\kappa_1=\eta_1}^{\nu_1}\sum_{\kappa_2=\eta_2}^{\nu_2}\cdots\sum_{\kappa_N=\eta_N}^{\nu_N}. \tag{14}$$

Substituting $m=1$ in (13), which corresponds to Rayleigh fading, we obtain [1, eq. (10)], as expected.

III. BER of $S+N$ SC Model 2 in Nakagami-m Fading

In this section, we derive the BEP of binary orthogonal signaling with $S+N$ SC Model 2. The receiver structure is depicted in [4, Fig. 3] and is not repeated here for the sake of brevity. Note that this receiver structure is limited to binary signaling only and therefore, we consider only binary NCFSK in this section. The receiver bases its decision on the RVs $W_i = X_{i1} - X_{i2}$ where X_{i1} and X_{i2} are expressed in (1) and (2), respectively with E_s being replaced with E_b.

It is shown in [6] that in i.n.d and i.i.d fading the BEP can be written as

$$P_b = \sum_{i=1}^{N}\int_0^\infty f_{W_i}(-w)\prod_{\substack{j=1\\j\neq i}}^{N}\left(F_{W_j}(w)-F_{W_j}(-w)\right)dw \tag{15}$$

and

$$P_b = N\int_0^\infty f_{W_1}(-w)\left(F_{W_1}(w)-F_{W_1}(-w)\right)^{N-1}dw \tag{16}$$

respectively. It can be shown that the CDF of W_i is given by

$$F_{W_i}(w) = \begin{cases} 1-\sum_{k=0}^{m_i-1}\sum_{l=0}^{k}\Upsilon_{k,l}(m_i,\bar{\gamma}_i)\exp\left(\frac{w}{2\sigma^2}\right) \\ \times\Gamma\left(l+1,\left(\frac{m_i}{m_i+\bar{\gamma}_i}+1\right)\frac{w}{2\sigma^2}\right), & w>0 \\ \exp\left(\frac{w}{2\sigma^2}\right)\Phi(m_i,\bar{\gamma}_i), & w<0 \end{cases} \tag{17a}$$

where $\Gamma(\cdot,\cdot)$ is the incomplete Gamma function defined as [11, eq. (8.352.2)],

$$\Gamma(n+1,z) = n!\exp(-z)\sum_{k=0}^{n}\frac{z^k}{k!}, \quad n\in\mathbb{N} \tag{17b}$$

and for notational simplicity we have defined $\Upsilon_{k,l}(m_i,\bar{\gamma}_i)$ and $\Phi(m_i,\bar{\gamma}_i)$ as

$$\Upsilon_{k,l}(m_i,\bar{\gamma}_i) = \frac{\Theta_{k,l}(m_i,\bar{\gamma}_i)}{\left(1+\frac{m_i}{m_i+\bar{\gamma}_i}\right)^{l+1}} \tag{17c}$$

$$\Phi(m_i,\bar{\gamma}_i) = 1-\sum_{k=0}^{m-1}\sum_{l=0}^{k}\Upsilon_{k,l}(m_i,\bar{\gamma}_i)\Gamma(l+1) \tag{17d}$$

where $\Gamma(n)=\Gamma(n,0)$. From the CDF of W_i, we derive the PDF of W_i by differentiation as

$$f_{W_i}(w) = \frac{1}{2\sigma^2}\exp\left(\frac{w}{2\sigma^2}\right)\Phi(m_i,\bar{\gamma}_i), \quad w<0. \tag{18}$$

The pdf is given just for $w<0$, as we only need the pdf for negative arguments for the calculation of the BEP in (15) and (16). Now substituting (17) and (18) in (15), we obtain the BEP of binary orthogonal NCFSK with $S+N$ SC in Nakagami-m fading as

$$P_b = \sum_{i=1}^{N}\Phi(m_i,\bar{\gamma}_i)\int_0^\infty \exp(-x) \times\prod_{\substack{j=1\\j\neq i}}^{N}\left[1-\sum_{k=0}^{m_j-1}\sum_{l=0}^{k}\Upsilon_{k,l}(m_j,\bar{\gamma}_j)\exp(x) \times\Gamma\left(l+1,\left(\frac{m_j}{m_j+\bar{\gamma}_j}+1\right)x\right)-\Phi(m_j,\bar{\gamma}_j)\exp(-x)\right]dx. \tag{19}$$

For $m_i=1$, $i=1,\cdots,N$, corresponding to i.n.d Rayleigh fading, (19) reduces to [6, eq. (26)], as expected. For i.i.d Nakagami-m fading the BEP becomes

$$P_b = N\Phi(m,\bar{\gamma})\int_0^\infty \exp(-x)\Big[1-\sum_{k=0}^{m-1}\sum_{l=0}^{k}\Upsilon_{k,l}(m,\bar{\gamma})\exp(x) \times\Gamma\left(l+1,\left(\frac{m}{m+\bar{\gamma}}+1\right)x\right)-\Phi(m,\bar{\gamma})\exp(-x)\Big]^{N-1}dx. \tag{20}$$

Note that for i.i.d Rayleigh fading, (20) reduces to [4, eq. (1)], as expected.

IV. SER of M-ary NCFSK with Classical Selection Combining

In classical SC, the receiver chooses the branch with the largest SNR. The SER of M-ary NCFSK in classical SC with N branch i.n.d fading can be written as

$$P_s = \sum_{n=1}^{M-1}\sum_{i=1}^{N}\frac{(-1)^{n+1}\binom{M-1}{n}}{n+1} \times\int_0^\infty \exp\left(-\frac{nx}{n+1}\right)f_{\gamma_i}(x)\prod_{\substack{j=1\\j\neq i}}^{N}F_{\gamma_j}(x)dx \tag{21}$$

where $f_{\gamma_i}(x)$ and $F_{\gamma_i}(x)$ are the PDF and CDF of the instantaneous SNR on the ith diversity branch and in Nakagami-m fading are given by [9]

$$f_{\gamma_i}(x) = \left(\frac{m_i}{\bar{\gamma}_i}\right)^{m_i}\frac{1}{\Gamma(m_i)}\exp\left(-\frac{m_i x}{\bar{\gamma}_i}\right)x^{m_i-1} \tag{22}$$

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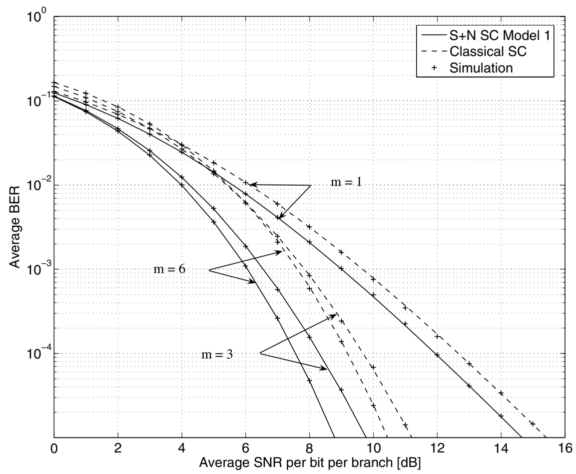

Fig. 1. The average BER of 4-ary orthogonal NCFSK for 4-branch $S+N$ SC Model 1 and classical SC in i.i.d Nakagami-m fading with $m = 1, 3$ and 6.

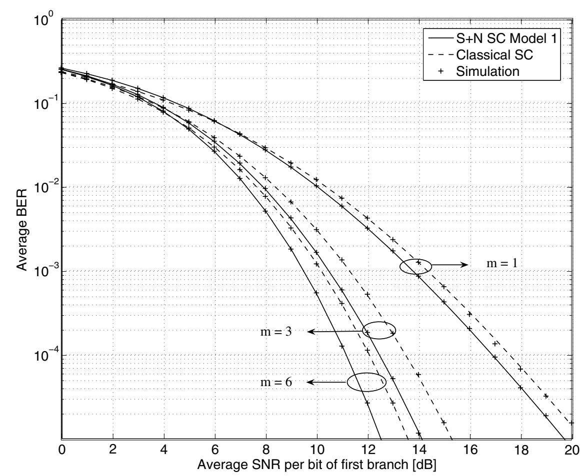

Fig. 2. The average BER of 4-ary orthogonal NCFSK for 4-branch $S+N$ SC Model 1 and classical SC in i.n.d Nakagami-m fading with $m = 1, 3$ and 6 and $\beta = 0.8$.

and

$$F_{\gamma_i}(x) = 1 - \frac{\Gamma\left(m_i, \frac{m_i x}{\bar{\gamma}_i}\right)}{\Gamma(m_i)} \tag{23}$$

respectively. Substituting (22) and (23) in (21), we obtain the SER of M-ary NCFSK with classical SC in i.n.d Nakagami-m fading as

$$P_s = \sum_{n=1}^{M-1} \sum_{i=1}^{N} \frac{(-1)^{n+1}\binom{M-1}{n}}{n+1} \frac{\left(\frac{m_i}{\bar{\gamma}_i}\right)^{m_i}}{\Gamma(m_i)} \int_0^\infty x^{m_i - 1} \times \exp\left(-\left(\frac{m_i}{\bar{\gamma}_i} + \frac{n}{n+1}\right)x\right) \prod_{\substack{j=1 \\ j \neq i}}^{N} \left(1 - \frac{\Gamma\left(m_j, \frac{m_j x}{\bar{\gamma}_j}\right)}{\Gamma(m_j)}\right) dx. \tag{24}$$

For i.n.d Rayleigh fading, one can show that (24) reduces to a closed-form expression given in [6, eq. (37)], as expected.

For i.i.d Nakagami-m fadings with integer values of m, using the multinomial theorem and after some mathematical simplification and integral evaluation, (24) reduces to a closed-form expression as

$$P_s = N \sum_{n=1}^{M-1} \frac{(-1)^{n+1}\binom{M-1}{n}}{n+1} \frac{\left(\frac{m}{\bar{\gamma}}\right)^{m}}{\Gamma(m)} \cdot \sum_{l=0}^{N} (-1)^l \binom{N}{l} \times \sum_{k=0}^{l(m-1)} \delta_{kl} \left(\frac{m}{\bar{\gamma}}\right)^k \frac{\Gamma(m+k)}{\left(\frac{m}{\bar{\gamma}} + \frac{n}{n+1} + \frac{lm}{\bar{\gamma}}\right)^{m+k}} \tag{25a}$$

where δ_{kl} can be computed using [3]

$$\delta_{kl} = \sum_{i=k-m+1}^{k} \frac{\delta_{i(l-1)}}{(k-i)!} I_{[0,(l-1)(m-1)]}(l) \tag{25b}$$

where $\delta_{00} = \delta_{0l} = 1$, $\delta_{k1} = \frac{1}{k!}$, $\delta_{1l} = l$ and $I_{[a,b]}(x)$ is defined as

$$I_{[a,b]}(x) = \begin{cases} 1, & a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}. \tag{25c}$$

For i.i.d fading (25) reduces to [1, eq. (6)], as expected.

V. Numerical Examples and Discussions

In this section, some numerical examples are presented to compare the performances of the $S+N$ SC receivers with those of classical SC and SLC in i.i.d and i.n.d Nakagami-m fading. For i.n.d fading we assume an exponentially decaying multipath intensity profile (MIP), i.e., $\bar{\gamma}_i = \bar{\gamma}_1 \exp(-\beta(i-1)), i = 1, \cdots, N$ where β is the average power decay factor. We present examples to compare the performances of the systems in i.i.d and i.n.d fading. Monte Carlo simulation results are also presented to test the validity of our analytical expressions.

We begin with the effect of fading parameter m on the system performance for both i.i.d and i.n.d fading. Figs. 1 and 2 show the average BER of 4-ary orthogonal NCFSK with 4 diversity branches in i.i.d and i.n.d Nakagami-m channels, respectively, where $m = 1, 3$ and 6 and $\beta = 0.8$. Both figures indicate that in a less faded environment and for a given BER, the SNR difference between $S+N$ SC Model 1 and classical SC increases. For example in Fig. 1, and at a average BER of 10^{-4}, the SNR difference is 0.72 dB, 1.37 dB and 1.63 dB for $m = 1$, $m = 3$ and $m = 6$, respectively. Comparing Fig. 2 with Fig. 1, one can see that in i.n.d fading, in contrast to the i.i.d case, the performance of $S+N$ SC Model 1 is inferior to that of classical SC when the average SNR is small. The differences are small, however, and they occur for large values of error rate, so that the $S+N$ Model 1 receiver will be preferred for practical implementations. Figs. 3 and 4 compare the effect of modulation order, M, on the relative performances of $S+N$ SC Model 1 and classical SC in i.i.d and i.n.d fading, respectively. While Fig. 3 shows that $S+N$

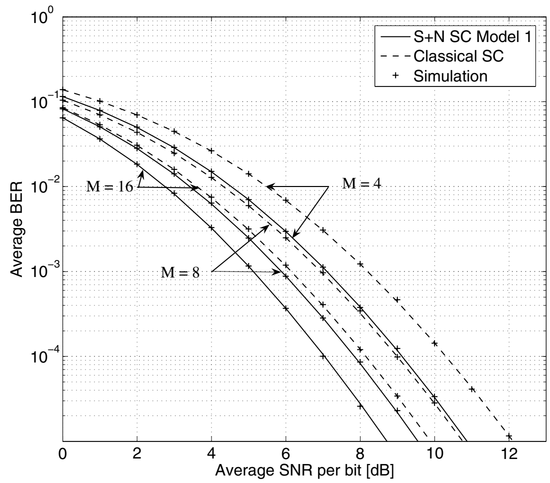

Fig. 3. The average BER of M-ary orthogonal NCFSK for 4-branch $S+N$ SC Model 1 and classical SC in i.i.d Nakagami-m fading with $m = 2$ for $M = 4, 8$ and 16.

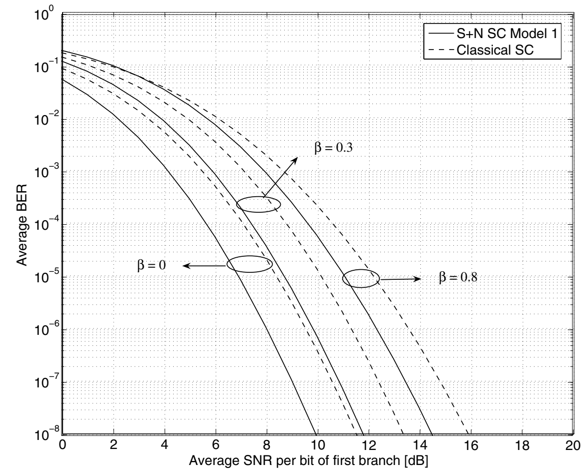

Fig. 5. The average BER of 16-ary orthogonal NCFSK for 4-branch $S+N$ SC Model 1 and classical SC in i.n.d Nakagami-m fading with $m = 4$ and $\beta = 0, 0.3$ and 0.8.

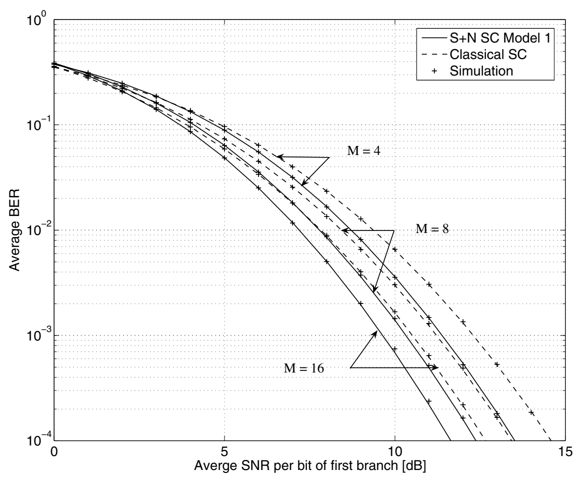

Fig. 4. The average BER of M-ary orthogonal NCFSK for 4-branch $S+N$ SC Model 1 and classical SC in i.n.d Nakagami-m fading with $m = 2$ and $\beta = 0.8$ for $M = 4, 8$ and 16.

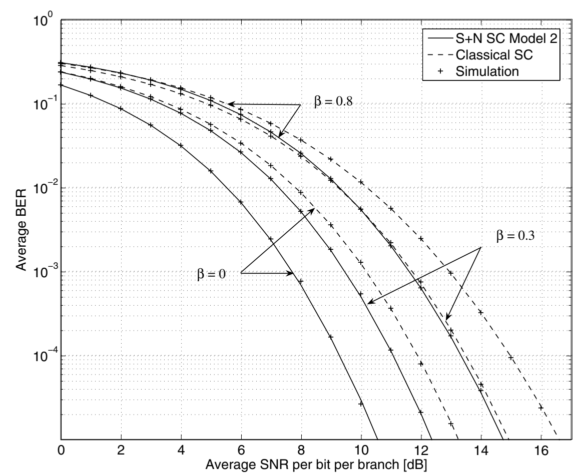

Fig. 6. The average BER of binary orthogonal NCFSK for 4-branch $S+N$ SC Model 2 and classical SC in i.n.d Nakagami-m fading with $m = 5$ and $\beta = 0, 0.3$ and 0.8.

SC Model 1 outperforms classical SC for the range of SNR given, we see in Fig. 4 that similar to Fig. 2, classical SC has a superior performance over $S + N$ SC Model 1 for small values of SNR. Fig. 4 also shows that the value of SNR at which the error rate curves of classical SC and $S + N$ SC Model 1 cross decreases as M increases. For example, the cross-over value is 3.4 dB, 2.7 dB and 2.3 dB for $M = 4, 8$ and 16, respectively.

In Fig. 5, the effect of the average power delay factor, β, on the relative performances of $S + N$ SC Model 1 and classical SC is examined, where the BER of 16-ary orthogonal NCFSK is plotted as a function of SNR per bit on the first diversity branch for 4-branch $S + N$ SC Model 1 and classical SC in Nakagami-m fading with $m = 4$. Notice that as β increases, the performance gain of $S + N$ SC Model 1 over classical SC decreases. Also it is important to notice that for the small region of SNR and for large β classical SC performs better that $S+N$ SC Model 1. This is because for larger β, we get weaker diversity branches in terms of signal strength and the noise in the system becomes dominant. Therefore, by choosing the largest output of square-law detectors, the $S + N$ SC receiver is in fact choosing a branch mostly affected by noise resulting in an inferior performance compared to classical SC. The same phenomena is present in Fig. 6, where for large β and in the small SNR region classical SC outperforms $S + N$ SC Model 2, while in the moderate to high SNR region $S + N$ Model 2 outperforms classical SC.

Fig. 7 shows the average BER performance of 8-ary NCFSK with 3, 4 and 5 branch diversity for $S + N$ SC Model 1, classical SC and SLC in i.i.d Nakagami-m fading. For SLC,

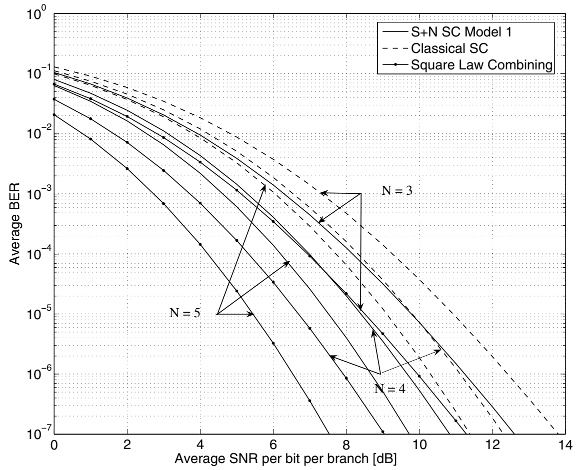

Fig. 7. The average BER of 8-ary orthogonal NCFSK in N-branch i.i.d Nakagami-m fading for $S+N$ SC Model 1, classical SC and SLC for $m = 3$ and $N = 3, 4$ and 5.

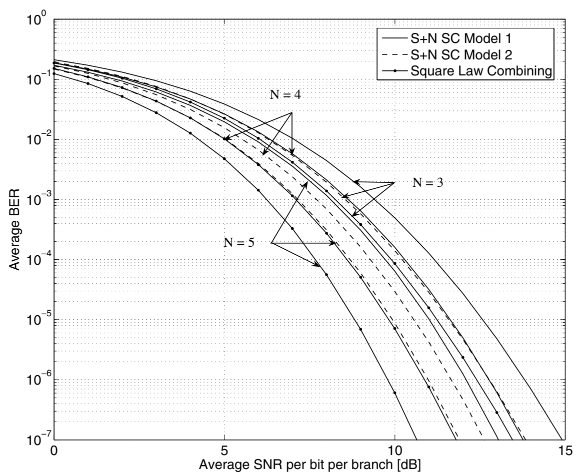

Fig. 8. The average BER of binary orthogonal NCFSK in N-branch i.i.d Nakagami-m fading for $S + N$ SC Model 1, $S + N$ SC Model 2 and SLC for $m = 5$ and $N = 3, 4$ and 5.

we have used the analytical expressions given in [12] to plot the curves in Fig. 7. Fig. 7 shows that as we increase the number of diversity branches, the performance gap between $S + N$ SC Model 1 and SLC increases. For example at an average BER of 10^{-3}, the SNR difference between $S+N$ SC Model 1 with SLC is 1.18 dB, 1.57 dB and 1.9 dB for $N = 3, 4$ and 5, respectively. The performance gap between $S + N$ SC Model 2 and SLC also increases as the number of diversity branches increases, as illustrated in Fig. 8. For example, at an average BER of 10^{-3} the SNR difference between the two systems is 0.29 dB, 0.57 dB and 0.92 dB, for $N = 3, 4$ and 5, respectively.

VI. CONCLUSION

This paper has analyzed the performances of M-ary orthogonal NCFSK with two $S + N$ SC receivers in i.n.d and i.i.d Nakagami-m (m, an integer) fading. For each receiver, tractable, analytical SER and BER expressions were obtained for the performance of the receiver in the presence of slow flat Nakagami-m fading. In addition, the performance of M-ary orthogonal NCFSK was analyzed with a classical SC receiver in Nakagami-m fading. Our results indicate that the performance of $S + N$ SC relative to that of classical SC is a function of the channel conditions. For example, we showed that in an i.n.d Nakagami-m fading environment with an exponentially decaying MIP, classical SC outperforms $S + N$ SC in the small SNR region and for large decay factor. The performance of $S + N$ SC receivers were also compared with that of SLC and it was observed that the performance gap increases as the number of diversity branches are increased.

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