

1. Three coins are tossed. Write out the sample space  $S$  of all outcomes. Find the probability that all three coins match. Next, find the probability that exactly two of the three coins match. (10 points)

**Solution:** The sample space consists of the 8 outcomes

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}.$$

$$p(\text{all three coins match}) = \frac{2}{8} = \frac{1}{4} \text{ and } p(\text{exactly 2 match}) = \frac{6}{8} = \frac{3}{4}.$$

2. If two dice are rolled, find the probability that the sum of the dice is 9. (10 points)

**Solution:** There are 4 ways two dice can add to 9 ( (3,6), (4,5), (5,4), and (6,3)), so  $p(\text{sum}=9) = \frac{4}{36} = \frac{1}{9} \approx 0.11$ .

3. Find the probability of being dealt all cards in the same suit in a five-card hand. Express your answer as a decimal number. (10 points)

**Solution:** To count the number of such hands, we first select a suit (4 ways) and then we select 5 cards from the selected suit ( ${}_{13}C_5$  ways). So  $p(\text{all in same suit}) = \frac{4 \cdot {}_{13}C_5}{{}_{52}C_5} = \frac{4 \cdot 1287}{2,598,960} = \frac{5148}{2,598,960} \approx 0.002$ .

4. We play a lottery in which three numbers in the range 1 through 12 are selected. Find the probability of winning this lottery, i.e., the probability of picking the three correct numbers. Then find the probability of picking exactly two of the three correct numbers. (10 points)

**Solution:**

$$p(\text{selecting all 3 correct numbers}) = \frac{{}_3C_3}{{}_{12}C_3} = \frac{1}{220} \approx 0.005.$$

$$p(\text{selecting 2 of the 3 correct numbers}) = \frac{{}_3C_2 \cdot {}_9C_1}{{}_{12}C_3} = \frac{27}{220} \approx 0.123.$$

5. You and two of your friends decide to play a game. Each of you flips a coin simultaneously. If all three coins match, each of your friends pays you \$10. If the coins don't all match, you pay each of your friends \$4. What is the expected value of this game from your point of view? Would this be a profitable game for you to play repeatedly? (10 points)

**Solution:**

Expected Value =  $p(\text{all 3 match}) \cdot \$20 + p(\text{not all match}) \cdot (-\$8) = \frac{1}{4} \cdot \$20 + \frac{3}{4} \cdot (-\$8) = -\$1$  so the game would not be profitable for you.

6. In a survey of people concerning their preference of cranberry sauce used in a traditional American Thanksgiving meal, we find that 125 of the men prefer jellied cranberry sauce, 20 men prefer whole-berry sauce, and 5 men have no preference, whereas, 160 of the women prefer whole-berry sauce, 30 prefer jellied sauce, and 10 have no preference. Find the probability that a randomly selected person in the survey prefers whole-berry cranberry sauce. Then find the probability that a person from the survey is a man, given that the person prefers jellied cranberry sauce. (10 points)

**Solution:**

Its a good idea to make a table to help on this.

	Jellied	Whole-berry	No Pref.
Men	125	20	5
Women	30	160	10
Totals	155	180	15

$$p(\text{a person prefers whole-berry}) = \frac{180}{350} \approx 51.4\%$$

$$p(\text{a person is male} | \text{prefers jellied}) = \frac{125}{155} \approx 80.6\%$$

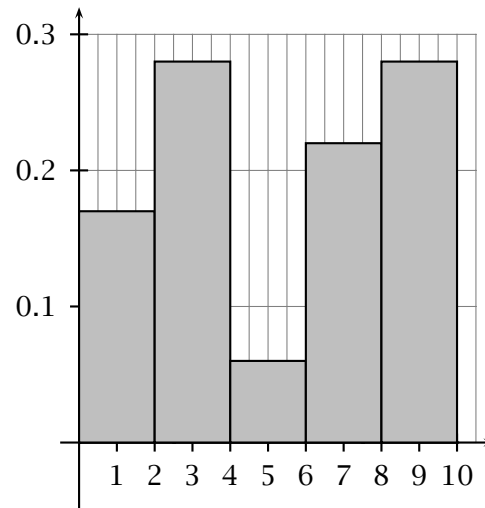
7. In a roll of two dice, let  $A$  be the event that the sum of the two dice is 6, and let  $B$  be the event that one of the dice is a 4. Are events  $A$  and  $B$  independent? Are they mutually exclusive? Explain. (10 points)

**Solution:**  $p(A) = \frac{5}{36}$ ,  $p(B) = \frac{11}{36}$ , and  $p(A \cap B) = \frac{2}{36}$ . So  $p(A|B) = \frac{p(A \cap B)}{p(B)} = \frac{2}{11}$ .  $A$  and  $B$  are not independent (i.e., they are dependent) because  $p(A) \neq p(A|B)$ . Since  $A \cap B \neq \emptyset$ , the events are not mutually exclusive.

8. Draw a *relative frequency histogram* for the dataset {1.1, 1.5, 2.3, 5.0, 9.1, 1.2, 7.4, 8.9, 7.1, 3.2, 7.2, 8.7, 2.9, 3.1, 7.9, 2.1, 8.1, 9.9}. Use 5 data groups each of width 2 starting at 0 (so that  $0 \leq x < 2$  describes the first category). (10 points)

**Solution:**

category	frequency	relative frequency
$0 \leq x < 2$	3	$\frac{3}{18} \approx 0.17$
$2 \leq x < 4$	5	$\frac{5}{18} \approx 0.28$
$4 \leq x < 6$	1	$\frac{1}{18} \approx 0.06$
$6 \leq x < 8$	4	$\frac{4}{18} \approx 0.22$
$8 \leq x \leq 10$	5	$\frac{5}{18} \approx 0.28$
Total	18	



9. Calculate the mean and median of the data set  $S = \{13, 12, 25, 13, 17, 14, 21, 34, 12, 17\}$  (10 points)

**Solution:**

$$\bar{x} = \frac{\sum x}{n} = \frac{178}{10} = 17.8 \quad \text{and} \quad \text{median} = \frac{14 + 17}{2} = 15.5$$

10. Find the variance and standard deviation for the data set  $S = \{2, 3, 5, 8\}$ . (10 points)

**Solution:**

$x$	$x - \bar{x}$	$(x - \bar{x})^2$
2	-2.5	6.25
3	-1.5	2.25
4	0.5	0.25
8	3.5	12.25
18	0	21

So  $s^2 = \frac{21}{3} = 7$  and  $s = \sqrt{7} \approx 2.65$ .