MA 110-91 §3.1 – 4.2	**Test #2**	*score*	**Name:** ____________ **13 April 2002**

1. Two dice are rolled. Write out the complete sample space and use it to find the probability that two dice differ by more than 2. Explain. *(10 points)*

   Solution:

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

   $p(\text{dice differ by more than 2}) = \frac{12}{36} \approx 0.33$

2. Four fair coins are tossed. Find the probability that there are more heads than tails in the result. Explain. *(10 points)*

   Solution: For there to be more heads than tails, there would have to be 3 heads and 1 tail, or all heads. The are 4 ways the former can occur and only 1 way the latter can occur. So out of the $2^4 = 16$ outcomes of tossing 4 coins, $p(\text{more heads than tails}) = \frac{5}{16} \approx 0.31$.

3. Find the probability of being dealt four cards from the same kind in a five-card hand. Explain your counting methods. Express your answer as a decimal number. *(10 points)*

   Solution: First, choose a kind (${}_{13}C_4$), then choose all 4 cards from the selected kind (${}_4C_4$), then select the fifth card from the 48 remaining (${}_{48}C_1$). So,

$$p(\text{4-of-a-kind}) = \frac{{}_{13}C_1 \cdot {}_4 C_4 \cdot {}_{48} C_1}{{}_{52}C_5} = \frac{13 \cdot 1 \cdot 48}{2,598,960} \approx 0.00024$$

4. If three dice are rolled, find the probability that there is at least one six among the three results. Explain. *(10 points)*

   Solution: We count the complementary event where we have no sixes.

$$p(\text{at least one six}) = 1 - p(\text{no six}) = 1 - \frac{5^3}{6^3} = 1 - \frac{125}{216} = \frac{91}{216} \approx 0.42$$

5. We play a lottery in which four numbers in the range 1 through 15 are selected. Find the probability of winning this lottery, i.e., the probability of picking the four correct numbers. Then find the probability of picking exactly three of the four correct numbers. Explain. *(10 points)*

   Solution:

$$p(\text{all four winning numbers}) = \frac{1}{{}_{15}C_5} = \frac{1}{1365} \approx 0.00073$$

$$p(\text{three of the four winning numbers}) = \frac{{}_4C_3 \cdot {}_{11} C_1}{{}_{15}C_5} = \frac{44}{1365} \approx 0.0032$$

6. You and two of your best math friends decide to play a game. Each of you flips a coin simultaneously. If all three coins match, each of your friends pays you \$5. If they don't all match, you pay each of your friends \$2. What is the expected value of this game from your point of view? Would this be a profitable game for you to play repeatedly? *(10 points)*

   Solution: The expected value is

   $$p(\text{all heads or all tails}) \cdot (\$10) + p(\text{both heads and tails occur}) \cdot (-\$4) = -\$0.50$$

   So the game is not in your favor; you should expect to lose an average of \$0.50 per game.

7. Two cards are selected. Let E_1 denote the event that the first card is a heart, and let E_2 denote the event that the second card is a heart. Are E_1 and E_2 mutually exclusive? Are they independent? Explain. *(10 points)*

   Solution: The events are not mutually exclusive since $E_1 \cap E_2$ contains the outcomes with a heart on both selections. The events are not mutually exclusive either since $p(E_2) = \frac{13}{52}$ whereas $p(E_2|E_1) = \frac{12}{51}$.

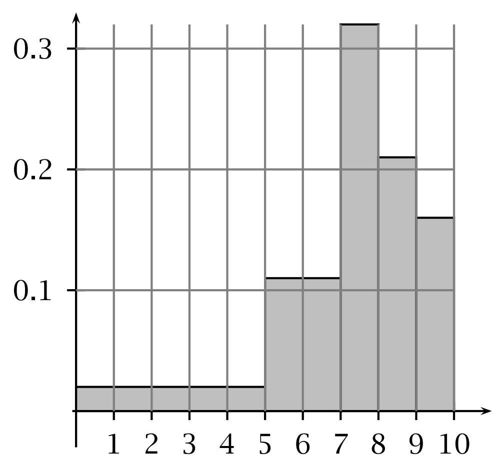
8. An electronics manufacturer buys its chips from three factories: A, B, and C. Factory A supplies half of the total chips while B and C each supply one-quarter. If the defective rates of the chips from the three factories are 1%, 2%, and 3%, respectively, find the probability that a randomly selected chip is defective. *(10 points)*

   Solution: Let E_x denote the event that a chip was manufactured at factory x and let E denote the event that a chip is defective. Then $p(E) = p(E|E_A) \cdot p(E_A) + p(E|E_B) \cdot p(E_B) + p(E|E_C) \cdot p(E_C) = 0.50 \cdot 0.01 + 0.25 \cdot 0.02 + 0.25 \cdot 0.03 = 0.0175$

9. Draw a *relative frequency density histogram* for the dataset {3.8, 4.5, 5.3, 5.6, 6.2,6.6, 7.2, 7.4, 8.9, 7.1, 9.2, 8.2, 7.7, 7.9, 8.1, 7.9, 8.1, 9.4, 9.9}. Use 5 data groups (bins): $0 \le x < 5$, $5 \le x < 7$, $7 \le x < 8$, $8 \le x < 9$, and $9 \le x \le 10$. *(10 points)*

   Solution:

category	freq.	relative freq.	density
$0 \le x < 5$	2	$\frac{2}{19}$	$\frac{2}{19\cdot 5} \approx 0.02$
$5 \le x < 7$	4	$\frac{4}{19}$	$\frac{4}{19\cdot 2} \approx 0.11$
$7 \le x < 8$	6	$\frac{6}{19}$	$\frac{6}{19\cdot 1} \approx 0.32$
$8 \le x < 9$	4	$\frac{4}{19}$	$\frac{4}{19\cdot 1} \approx 0.21$
$9 \le x \le 10$	3	$\frac{3}{19}$	$\frac{3}{19\cdot 1} \approx 0.16$
Total	19		



10. Calculate the mean and median of the data set $S = \{8, 3, 12, 17, 11, 14, 9, 7, 5, 13\}$ *(10 points)*

    Solution: Order the data we get 3, 5, 7, 8, 9, 11, 12, 13, 14, 17. So the median of the 10 data points is halfway between items 5 and 6: $\frac{9+11}{2} = 10$. The mean is given by

    $$\bar{x} = \frac{\Sigma x_i}{10} = \frac{99}{10} = 9.9$$

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