

Math 112 Carter Test 2 Spring 2002

General Instructions: Write your name on only the outside of your blue book. Put your test paper inside your blue book as you leave. Solve each of the following problems. Point values (out of 100 points) are indicated on the problems.

1. For each of the following systems of equations (1) determine if there is a solution; (2) if there is a solution, write it in standard form. That is: in case there are an infinite number of solutions write the standard form of the solution set. If there is one solution, write that explicitly. (*8 points each*)

(a)

$$x + 2y = 6$$

$$3x - y = 1$$

(b)

$$x + y + z = 1$$

$$2x - 2y + z = 1$$

$$x + y - z = 4$$

(c)

$$a + b + c + d = 1$$

$$2a - b + c + d = 2$$

$$a + 3b + 2c - d = 3$$

2. (6 points) The reduced row-echelon form of a system of 5 equations in 6-unknowns  $(a, b, c, d, e, f)$  is given as

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & -2 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Does the system have a solution? If so, write it in standard form.

3. (10 points each) Solve the inequality (10 points each):

(a)  $x - 2 < \frac{x-2}{x-1}$

(b)  $x(x - 1) + (x - 1) < 0$

(c)  $0 \leq \frac{x}{x^2-16}$

(d)  $|3x - 4| < 15$

4. Give a polynomial with real coefficients (in factored form) that has roots and degree indicated. (5 points each)

(a) Roots:  $2 - i$ , and 3, degree = 3

(b) Roots:  $-i$ , 2, and 2, degree = 4

5. (10 points) Factor  $f(x) = x^4 - 3x^3 - 6x^2 + 6x + 8$  over the real numbers.

6. Use synthetic division to find the quotient and remainder. (5 points each)

(a)  $(x^5 - 243)/(x - 3)$

(b)  $(2x^4 - 7x^3 + x^2 - 2)/(x - 3)$