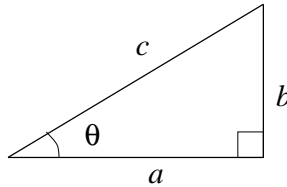


Math 113 Carter Final Exam Fall 2003

General Instructions: Write your name on only the outside of your blue book. Put your test paper inside your blue book as you leave. Do all of your work and write your solutions inside your blue book. Do not write on this test sheet. Solve each of the following problems.

1. (5 points) Based on the labeled triangle below, indicate the definitions of the 6 trigonometric functions for the angle θ that is illustrated.



2. (5 points each) Convert between degrees and radians as indicated. Leave your answer as a rational number times π , or a positive whole number. Note some angles may be larger than a full rotation.

(a) $135^\circ = x \text{ rad}$

(b) $x^\circ = 7\pi/3 \text{ rad}$

(c) $9\pi/4 \text{ rad} = x^\circ$

(d) $90^\circ = x \text{ rad}$

3. Sketch the graphs of the following functions. (10 points each)

(a) $y = \sin(\pi x)$

(b) $y = 15 \cos(x - \pi/2)$

(c) $y = 2 \sin(x) + 1$

4. Verify the following identities: (5 points each)

(a) $\cot(x) + \csc(x) = \frac{\sin(x)}{\cos(x)-1}$

(b) $\cos^2(x) - \sin^2(x) = 2 \cos^2(x) - 1.$

5. Solve for all values of $x \in [0, 2\pi]$. Note the solution may be straight forward, or you may need a graphing calculator (*10 points each*).

(a) $4 \cos^2(x) = 3$.

(b) $x \sin(x) + 1 = 0$.

6. (*5 points*) Find all solutions in the complex plane to the equation, $z^5 = 1$. Express your answer in the form, $z = \cos(x) + i \sin(x)$ where x is a rational number times π .

7. The following complex numbers are given as a pair $[r, \theta]$ where r is the signed distance from the origin and θ is an angle. Determine their cartesian coordinates (*5 points each*).

(a) $[3, \pi/4]$

(b) $[-1, 7\pi/6]$

8. Solve the following triangles (in each of them (α, β, γ) are the respective angles opposite to the sides (a, b, c)) (*5 points each*):

(a) $a = 3, b = 4, c = 5$

(b) $\gamma = 90^\circ, c = 11, b = 8$.

(c) $a = 14, b = 12, \gamma = 42^\circ$.

9. Complete the square and sketch the graph (*10 points each*):

(a) $x^2 + 4x + y - 3 = 0$

(b) $2y^2 + x^2 - 8y + 4x + 3 = 0$

(c) $-y^2 + 4x^2 - 2y - 16x - 1 = 0$

10. (*5 points*) Parametrize the ellipse

$$\frac{(x - 3)^2}{36} + \frac{(y - 2)^2}{25} = 1$$

by a pair of functions $x = x(t)$, $y = y(t)$ that start from $(9, 2)$ and travels once counterclockwise in a length of 2π .