

Applied Calculus Final Exam

Do the problems in order in your bluebook. Show your work.

1. You decide to make Math T-shirts. You have fixed costs of \$166 and variable costs of \$5 per shirt. Find the cost equation. If you sell the shirts for nine dollars each, what is your break-even point ?
2. You are selling iced cappuccino's. If you charge one dollar, you end up making 100 sales every week. Each quarter increase in price results in 20 fewer sales. Find the equation of the demand curve and graph it.
3. The population of Podunk grows at an annual rate of 9.5%. If initially there were 200,000 inhabitants, how long before the population reaches one million ?
4. Set up but do not compute a Riemann sum with 4 rectangles using the right-hand endpoints for approximating the area bounded by $y = \cos(x)$ from $x = 0$ to $x = \pi$ (x in radians). Draw a graph showing everything.
5. Find the average value of $y = \sqrt{x}$ over $[1, 4]$ and show how it is represented graphically.
6. Find the derivative of $f(x) = 8x^5 \cdot e^{-x}$. What can you conclude about $f(x)$ at points where $f'(x) > 0$?
7. Find the second derivative of $g(x) = \ln(x^2 + 1)$. What can you conclude about $g(x)$ when $g''(x) > 0$?
8. You are driving slowly to your favorite class – Math 120, of course – when you realize that you have forgotten your bag lunch. You turn around and start to drive very fast back home, but a police car pulls you over. You manage to use Calculus to convince the policeman to refrain from giving you a ticket, though it does take you some time. You then drive slowly home where you very quickly get the lunch. Before you can leave to go to your Calculus class, your Mom tells you take your little sister to her Ballet class, which is in the opposite direction from school. You drive very fast to the Ballet class, teasing your sister and eating your lunch, and drop her off, practically without even coming to a complete stop. You then drive even faster towards school. You get halfway there (halfway as measured from the Ballet class) but your luck fails you as the very same policeman from earlier pulls you over for speeding. You start ranting and raving about Ballet, Calculus and chocolate-covered space aliens, when all of sudden and for no apparent reason, he takes you away for observation (we won't even speculate on why). Let $f(t)$ be the distance you are from your favorite place in the world: your Calculus class, where t is time with $t = 0$ being the time you start this adventure and $t = A$ being the time you are taken away (note that $f(t)$ is **not** the distance you have travelled). Sketch a graph of $f(t)$ for $0 \leq t \leq A$. Label the various sections of the graph.
9. A metallurgical engineering graduate student places a metal bar in the freezer for 45 minutes. Then he takes it out and puts it in a hot oven for 15 minutes. Then he puts it into a pitcher of ice water for 30 minutes. Let $T(t)$ be its temperature at time t . Sketch the graph of $T'(t)$ for $0 \leq t \leq 90$.
10. Suppose a density function has graph for $0 \leq x \leq 10$ and that, other than the scale on the vertical axis, looks like two pieces, both straight lines. The first piece is a straight line of slope $m = 1$ emanating from the origin and ending at the point $(4, 4)$. The next piece is a horizontal line of height 4 that starts at the point $(4, 4)$ and ends at $(10, 4)$. Find the median of the quantity described by the density function. Explain.
11. A mouse is placed at left end of a 20 foot long tube. Over the next 109 minutes, a graduate student lab assistant exposes the mouse to cheese and incredibly bad renditions of old Nirvana tunes, making the mouse run back and forth across the tube thirty-one times to either try to get the cheese or try to escape from the music. In the end, the mouse passes out at the right end of the tube. Let $v(t)$ be the mouse's velocity at time t in feet per minute, where positive means it is going towards the right and negative means it is going towards the left. Find $\int_0^{109} v(t) dt$ and explain and justify your answer. (Note: the lab assistant wrote up his findings and now teaches at a university, spending the occasional night out at Karaoke bars – just kidding)