Fall 00 **Math 125 Exam 1 Review** Prof. Brick

1. How is the graph of $y = 9 + f(x - \sqrt{\pi})$ obtained from that of $y = f(x)$.

2. For what value of k is $f(x) = \begin{cases} 3x^2 + k - 1, & \text{if } x \le 2 \\ kx, & \text{if } x > 2. \end{cases}$ continuous at $x = 2$?

3. Find the following limits:

(a) $\lim_{x\to+\infty} \frac{3x - 4x^3}{5x^3 - 7}$ (b) $\lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{3x+5}$ (c) $\lim_{x\to 2} \frac{x-2}{x^3-8}$

4. Numerically approximate $\lim_{h\to 0} \left(\frac{\sin(h)}{h}\right)$. What does this have to do with a graph ?

5. Use three secant lines from for $y = e^{-x^2}$ to estimate the slope at $x = 0$. Graph it.

6. State the definition of the derivative and use it to find the derivatives of the following:

(a) $f(x) = \sqrt{2x+1}$ (b) $g(t) = 100 + 25t - 16t^2$ (c) $h(x) = \frac{1}{5x+1}$

7. For what values of x is $y = |x^2 - 1|$ not differentiable ? Why ?

8. Find a linear approximation for $\sqrt{x}$ and use it to approximate $\sqrt{15.9}$

9. Using $\frac{d}{dx}(\cos(x)) = \sin(x)$, find a linear approximation of $y = \cos(x)$ for x near 0.

10. Sketch a curve $y = f(x)$ with $f'(x) > 0$ for all x but with $f''(x)$ changing sign.

11. Suppose investing \$1000 for ten years at annual interest of $r\%$ compunded continuously yields a balance of $g(r)$ dollars. What does $g(5) = 1649$ and $g'(5) = 165$ tell you ?

12. Let $P(t)$ be the price of the stock MATH, where t is time. What does the statement "the stock is rising in price but leveling off" say about $P'(t)$ and $P''(t)$?

13. Use the Intermediate Value Theorem to prove that $x^2 = \sqrt{x+1}$ for some $1 < x < 2$.

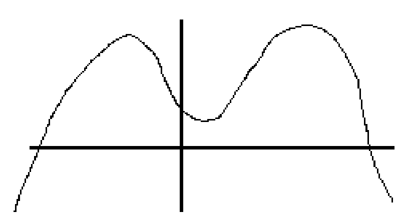
14. A ball is thrown up from a window. Its height after t seconds is $s(t) = 96 + 16t - 16t^2$. Using the fact that the derivative of position is velocity, find how fast it hits the ground ?

15. Use shortcuts to find the derivatives of the following functions:

(a) $f(x) = 15x^5 - 15\pi x + 15\pi^2$ (b) $g(x) = (3x^2 - 1)^2$ (c) $h(t) = \sqrt{5} - \sqrt{t} + \frac{5}{t^6}$

16. Find the tangent line to $f(x) = 3x^4 - 10x$ at $x = 1$.
Graph $f(x)$ and the tangent line.

17. Sketch a graph of $y = f'(x)$ given the graph of $f(x)$



18. Review all the quizzes, all the homework, and everything else.

MA 125, Fall 00 **Calculus 1 — Exam 1** Prof. Brick

Do the problems in order in your bluebook. Show your work.

1. For what value of k is the following function continuous at $x = 2$?

$$f(x) = \begin{cases} \frac{1}{2}x^2 + k, & \text{if } x \leq 2; \\ 5x - k, & \text{if } x > 2. \end{cases}$$

2. Using the derivative, approximate $\sqrt{15.96}$

3. Sketch a curve $y = f(x)$ with $f''(x) > 0$ for $x > 0$, with $\lim_{x \to -\infty} f(x) = -1$, with $\lim_{x \to -2^-} f(x) = +\infty$, with $\lim_{x \to -2^+} f(x) = -\infty$ and with $\lim_{x \to +\infty} f(x) = 1$.

4. Find the tangent line to $f(x) = 5x^2 - 8x$ at $x = 1$. Graph $f(x)$ and the tangent line.

5. A ball is thrown up from a window. Its height after t seconds is $s(t) = 96 + 16t - 16t^2$. When and at what velocity does it hit the ground ?

6. Find the derivative of $h(x) = \sqrt[3]{x^2} + e^2 - e^x + \dfrac{3}{x} + \pi^3$ (you may use the shortcuts).

7. Find $\displaystyle\lim_{x \to +\infty} \frac{150x - 3x^4}{5x^4 - 190x^3}$.

8. Use the definition (with limits) to find the derivative of $f(x) = \dfrac{1}{9x + 1}$

Fall 00 **Math 125 Exam 2 Review** Prof. Brick

1. Find the first derivatives of the following functions:
(a) $g(x) = \left(4x^6 + \sqrt{x^2+1}\right) \cdot (5x^2 + \pi^2 - e^\pi)$
(b) $f(x) = (e^{-x} + 5^x + e^2)^7$
(c) $h(t) = \dfrac{t^3 + 5t^2}{\cos(t) - 13t^6}$
(d) $\alpha(u) = \arctan(u^2 + 4)$
(e) $f(t) = \ln(\ln(\ln(t))) + \ln(\ln(\ln(\pi^8)))$
(f) $w(x) = (x^2+1)^{(x^2+1)}$
(g) $h(t) = e^{e^{e^{e^t}}}$

2. Find the line tangent to $xy^2 - 3x^2y = \sin(3x^2 - y)$ at $(1, 3)$

3. Find a formula for $\dfrac{dy}{dx}$ given that $x^2 + 3y^2 = \arctan(xy)$

4. Find a linear approximation for $\sqrt{x}$ and use it to approximate $\sqrt{15.9}$

5. Find a good linear approximation for $y = \dfrac{1}{1+x}$ near $x = 0$

6. Use the first derivative test to find the local min/max's of of the following functions. Also indicate where each function is increasing, and where each is decreasing.
(a) $y = x^4 + 2x^2 - 500$ (b) $y = 3x^4 - 8x^3$ (c) $y = (x^2 - 4x)^{\frac{2}{3}}$

7. Use the second derivative test to classify the the local min/max's of the following:
(a) $y = \ln(x^2+1)$ (b) $g(t) = \arctan(x^2+1)$ (c) $y = x^3 + 3x^2 - 9x$

8. A thirteen foot ladder is leaning up against a wall. The end of the ladder slips and slides down at a rate of 2 ft/sec. How fast is the other end moving away from the wall, when the top of the ladder is 5 feet off the ground ?

9. Two cars leave a town at the same time. One is travels north at 40 mph, the other west at 30 mphr. At what rate is the distance between them changing two hours later ?

10. A spherical balloon is being inflated at the rate of 5 cubic inches a minute. At what rate is the radius changing after ten minutes ?

11. A kite is flown at a constant height of 200 feet. Its speed is 6 ft/sec. At what rate in degrees/sec is the angle made by the string (with the horizontal) changing when 360 ft of string has been let out ?

12. A man 6 feet tall is walking away at a rate of 2 ft/sec from a streetlight that is 18 feet high. At what rate is the length of his shadow changing ?

13. Find the absolute min/max's of of the following functions over the indicated intervals:
(a) $y = \sin(x) + \cos(x)$ over $[0, \pi/3]$ (b) $y = x^3 - 12x$ over $[1, 5]$
(c) $y = 3xe^{-x}$ over $[0, 2]$

14. Redo all the homework and quizzes.

Fall 00 **Math 125 Final Exam Review** Prof. Brick

1. Go over all the quizzes, all the homework, and the previous review sheets.

2. Set up (but do not compute) a Riemann sum using 6 rectangles and the left hand endpoint rule for the area bounded by $y = e^{-x^2}$ from $x = 0$ to $x = 2$. Draw a graph showing the rectangles.

3. Use the fundamental theorem of calculus to compute $\displaystyle\int_1^2 (12x^2 - x + \pi)\, dx$

4. Without doing any computation, explain graphically why $\displaystyle\int_{-n}^{+n} x^3\, dx = 0$ for all $n \geq 0$.

5. Set up (but do not compute) a Riemann sum using 8 rectangles and the left hand endpoint rule that could be used to approximate $\ln(3)$.

6. Go over all the quizzes, all the homework, and the previous review sheets.

7. Use the fundamental theorem of calculus to compute $\displaystyle\int_1^2 (x^3 + \sqrt{x})\, dx$

8. Suppose $\displaystyle\int_a^b f(x)\, dx < 0$, what can you conclude about $f(x)$ over $a \leq x \leq b$?

9. A box is to be formed by cutting out square corners from a square piece of cardboard measuring 10 inches by 10 inches. How big of a corner should be cut to yield a box of maximum volume ?

10. A rectangular garden is to be fenced off along a long wall. The garden is to enclose 100 square feet. How should it be designed in order to use the least amount of fencing ?

11. A cylindrical can having a volume of 25 cubic inches is to be constructed using the least amount of material. Design the can.

12. Go over all the quizzes, all the homework, and the previous review sheets.

13. Find the area – not the signed area – bounded by $y = 4 - x^2$ from $x = 0$ to $x = 3$

14. Use Newton's method to find the solution to $\cos(x) = x$ to three decimal places.

15. Explain graphically how Newton's method works.

16. You decide to sell calculus cups on campus. If you charge \$ 5 each, you will sell 200 of them. Each dime increase results in five fewer sales. What price maximizes revenue ?

17. Suppose $C(x) = 100 + 8x + x^2$ is the cost function for producing widgets. Find the marginal and average cost. What production level minimizes average cost ?

18. Suppose cost is given by $C(x) = 1000 + 40x$ and the demand curve is given by the formula $p(x) = 80 - x$ (where p is price and x is the number sold). Maximize the profit.

19. Find $\displaystyle\lim_{x \to 0} \frac{e^x - x - 1}{x^2}$

20. Review everything else.

MA 125, Fall 00 **Calculus 1 — Final Exam** Prof. Brick

Do the problems in order in your bluebook. Show your work and justify your answers. Give exact answers whenever possible. You may use your calculator to compute numerical values only.

1. Using a linear approximation for $\sqrt{x}$ near $x = 9$, approximate $\sqrt{8.9}$.

2. Suppose $C(x) = 10000 + 85x - (0.02)x^2$ is the cost function for mining the rare mineral Mobilium, where x is in tons. Find the marginal and average cost.

3. A ball is dropped from a window. Its height in feet after t seconds is $s(t) = 160 - 16t^2$. Find when and how fast it hits the ground ?

4. Find $\lim_{x \to 0} \dfrac{e^{3x} - 3x - 1}{x \cdot \sin(x)}$

5. Find the equation of the line tangent to the curve $y^2 - x^2 - 5 = \tan(xy - 6)$ at $(2, 3)$.

6. Set up (but do not compute) a Riemann sum, call it R_4, using 4 rectangles and the right hand endpoint rule for the area bounded by $y = \dfrac{1}{1+x^2}$ from $x = 0$ to $x = 1$. Draw a graph showing the rectangles. Suppose you proceed to use more and more rectangles, say n rectangles with Riemann sum R_n. Compute $\lim_{n \to +\infty} R_n$.

7. For what value of k is $f(x) = \begin{cases} x^3 + k - 1, & \text{if } x \le 2 \\ kx, & \text{if } x > 2. \end{cases}$ continuous at $x = 2$?

8. Using the evaluation version of the fundamental theorem, find $\displaystyle\int_1^2 \left(2x^2 - 4 + \frac{1}{x}\right) dx$

9. Use the second derivative test to classify the the local min/max's of $y = 3x^3 - 4x$.

10. A cylindrical can (with top and bottom) having a volume of 54π cubic centimeters is to be constructed using the least amount of material. Design the can.

11. Find $\displaystyle\int (x^2 + 3)^2 \, dx$.

12. The TV show "Unnatural Experiences and Nauseating Aliens" claims to have photographed a UFO as it flew overhead at a constant altitude of 3 miles and at a constant speed. The show's hosts say that they took the photo at the instant after the UFO had already flown horizontally 4 miles past the point directly above them. They state that the radar gun showed that at the instant the photograph was taken, the distance between the photographer and the UFO was growing at a speed of 384 mph. Determine the speed at which the UFO was traveling.

13. You wish to use Newton's method to solve $x^5 + 3x - 1 = 0$. Your initial guess for the root is $x_1 = 0.5$. Draw a graph that illustrates how Newton's method gets x_2 from x_1. Calculate x_2.