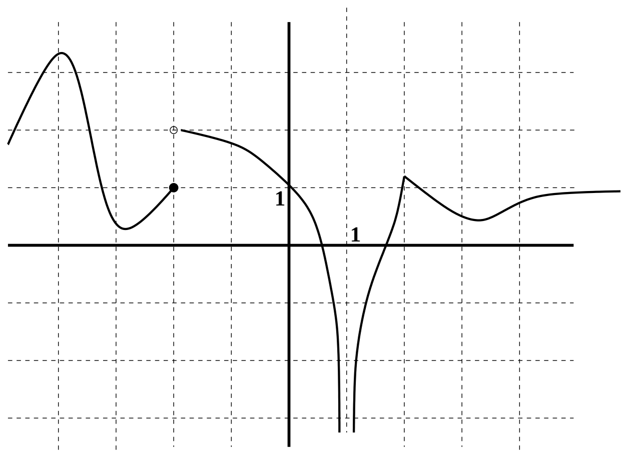
Math 125 - First Exam

1. (20 pts)
   (a) For $y = f(x)$, define the *derivative* of f at a, $f'(a)$.
   (b) What three things does the derivative do?

2. (35 pts) For the function whose graph is pictured below,



   (a) $\lim_{x\to 0} f(x) =$
   (b) $\lim_{x\to -2} f(x) =$
   (c) $\lim_{x\to -2^-} f(x) =$
   (d) $\lim_{x\to -2^+} f(x) =$
   (e) $\lim_{x\to \infty} f(x) =$
   (f) $f'(0) =$
   (g) $f''(4) =$
   (h) At what points is f NOT continuous?
   (i) At what points is f NOT differentiable?
   (j) On what intervals is $f''(x) > 0$?
   (k) Sketch (on the same picture) the graph of f'.

3. (20 pts) Let $f(t)$ be the temperature of a cup of coffee in degrees Fahrenheit, t minutes after it has been poured.
   (a) Interpret $f(4) = 120$ and $f'(4) = -5$. What are the units of each.
   (b) Estimate the temperature of the coffee after 5 minutes and after 8 minutes. What can you say about the accuracy of these estimates?

4. (25 pts) Find the following.
   (a) $\lim_{x\to 3} \dfrac{x^2+9}{x+3}$
   (b) $\lim_{x\to 3} \dfrac{x^2+9}{x-3}$
   (c) $\lim_{x\to 3} \dfrac{x^2-9}{x-3}$
   (d) Using the definition, $f'(3)$ for $f(x) = x^2$.
   (e) g' and g'' for g given by $g(t) = 144 + 96t - 16t^2$.

Math 125 - Second Exam

1. (30 pts) Find the derivative with respect to x
   (a) $y = x^3 - 2x + 1$
   (b) $f(x) = e^{-x^2}$

(c) $g(x) = \tan^{-1} x$
(d) $h(x) = \ln 3$
(e) $s = \sqrt{t^2 + 1}$
(f) $z = x \ln(x) - x$

2. (20 pts) Find intervals on which $F(x) = xe^{-x}$ is increasing, decreasing, concave up, concave down, local maxima and minima, and inflection points.

3. (15 pts) Find all points on the curve $x^2 + 2y^2 = 1$ where the slope of the tangent line is 1.

4. (10 pts) A plane flying with a constant speed of 300 km/h passes over a radar station at an altitude of 1 km and climbs at an angle of 30°. At what rate is the distance from the plane to the radar station increasing a minute later?

5. (15 pts) The hyperbolic sine and cosine are defined by $\sinh x = (e^x - e^{-x})/2$ and $\cosh x = (e^x + e^{-x})/2$ and other hyperbolic "trig" functions are defined analgous to corresponding trigonometric functions.
   (a) Show that $\cosh^2 x - \sinh^2 x = 1$.
   (b) Show that $\frac{d}{dx} \sinh x = \cosh x$. Find the derivative of cosh.

6. (10 pts) If the length of each edge of a cubical box is increased by 1% approximately what is the percent increase in the volume?

Math 125 - Third Exam

1. (40 pts) Evaluate the following:
   (a) $\lim_{z \to 0} \frac{\tan z}{z}$
   (b) $\lim_{t \to \infty} t^2 e^{-2t}$
   (c) $\int x^2 - 3 \sin x + 3\sqrt{x}\, dx$
   (d) $\lim_{\theta \to 0^+} \frac{\cos \theta}{\theta}$
   (e) $\int_1^3 (y-1)(y-2)\, dy$

2. (15 pts) A rock is dropped from the top of a cliff and hits the valley below in 5 seconds. Assuming air resistance in negligible, determine how high the cliff is.

3. (15 pts) An animal population is increasing at a rate of $200 + 50t$ per year (where t is measured in years). By how much does the animal population increase between the fourth and tenth years?

4. (15 pts) The velocity of a wave of length L in deep water is

$$v = K\sqrt{\frac{L}{C} + \frac{C}{L}}$$

where K and C are known positive constants. What is the length of the wave that gives the minimum velocity?

5. (15 pts) Use a calculator to make a table of values of the right Riemann sums, R_n for the integral $\int_0^\pi \sin x \, dx$ with $n = 5, 10, 50,$ and 100. What do these numbers appear to be approaching?

Math 125 - Final Exam

1. (20 pts) For $y = f(x)$,
   (a) Define the *derivative* of f at a, $f'(a)$.
   (b) What three things does the derivative do?
   (c) Define $\int_a^b f(x) \, dx$, the *integral* of f from a to b.
   (d) What things does the integral give you?

2. (30 pts) Find the following.
   (a) $\lim_{x \to 3^-} \frac{x^2 + 9}{x - 3}$
   (b) $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$
   (c) The derivative of $f(x) = xe^{-x^2}$
   (d) The derivative of $z = x \ln(x) - x$ with respect to x
   (e) $\lim_{t \to \infty} t^3 e^{-t}$
   (f) $\int_1^3 \frac{y^4 - 1}{y} \, dy$

3. (20 pts) Let $f(t)$ be the temperature of a cup of coffee in degrees Fahrenheit, t minutes after it has been poured.
   (a) Interpret $f(4) = 120$ and $f'(4) = -5$. What are the units of each.
   (b) Estimate the temperature of the coffee after 5 minutes and after 8 minutes. What can you say about the accuracy of these estimates?

4. (10 pts) If the population of the United States is growing at a rate $r(t) = 5(1 + .02t + .0002t^2)$ million people per year, how many more people will we have in 2012 than we do now? ($t = 0$ corresponds to now.)

5. (10 pts) The angle of elevation of the sun is decreasing at a rate of 0.25 rad/hr. How fast is the shadow cast by a 400 ft tall building increasing when the angle of elevation of the sun is $\pi/6$?

6. (10 pts) The velocity of a wave of length L in deep water is

$$v = K\sqrt{\frac{L}{C} + \frac{C}{L}}$$

where K and C are known positive constants. What is the length of the wave that gives the minimum velocity?