

Final

Instructor: Abhijit Champanerkar
Calculus IA, MA 125, Fall 2004

Date: Dec 9th 2004

Points: 100

Time: 2 hours

Name: _____

To receive full credit, you must explain your answers.

No calculators of any type are allowed.

Each question is worth 10 points.

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Total	

1. Find the following limits.

(a) (3 points) $\lim_{x \rightarrow 0} \frac{e^{7x} - e^{3x}}{2x}$

(b) (3 points) $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^2 - 7x + 10}$

(c) (4 points) $\lim_{x \rightarrow \infty} \frac{x^3 + 3x - 10}{e^{2x}}$

2. (a) (3 points) Let $y = \frac{\ln(2 + \cos x)}{1 + x^2}$. Find $\frac{dy}{dx}$.

(b) (4 points) Let $x \sin y = y^3 + x^3$. Find $\frac{dy}{dx}$.

(c) (3 points) Let $f(x) = e^{5x}$. Find $f^{(50)}(x)$.

3. (a) Let

$$f(x) = \begin{cases} 2ax^2 - 5 & \text{if } x < 2 \\ a + x & \text{if } x \geq 2 \end{cases}$$

Find the value of a if $f(x)$ is continuous at $x = 2$.

(b) Find the values of a and b if the curve $y = e^{ax} + bx^2 - 3x$ has a local maximum at $x = 0$ and $f''(0) = -1$.

4. (a) (6 points) Find the point on the curve $y = \sqrt{1 - x^2}$ where the tangent is parallel to the line $x + y + 4 = 0$. (Hint: There is only one point).

- (b) (4 points) Find $f(x)$ if $f''(x) = 4x^3 + \sin x$ and $f'(0) = 2$, $f(0) = 3$.

5. (a) Find the critical points of the function $f(x) = x^4 - 2x^2 + 10$ and classify them.

(b) The volume of a cube is increasing at the rate of $6 \text{ cm}^3/\text{min}$. Find the rate at which the surface area of the cube is increasing when the volume is 125 cm^3 .

6. Let $f(x) = \frac{x^2 + 3}{x + 1}$.

(a) (4 points) Find the horizontal and vertical asymptotes of $y = f(x)$.

(b) (6 points) Find the intervals of increase and decrease of f and the local maxima and minima of f .

7. For the function f in Problem 6 :

(a) (*6 points*) Find the intervals of concavity and inflection points of f .

(b) (*4 points*) Sketch the graph of f .

8. Find the point on the line $2x + y = 4$ that is closest to the point $(9, 1)$.

9. (a) (4 points) $\int_{-2}^0 \sqrt{4-x^2} dx$

(b) (6 points) When brakes are applied a car slows down with a constant deceleration of 4 ft/s^2 , producing skid marks measuring 200 ft before coming to a stop. How fast was the car traveling when the brakes were applied ?

10. Evaluate the following integrals.

(a) $\int_{-2}^1 (2|x| + 5)dx$

(b) $\int_0^3 (4 - 2x)dx$

BONUS QUESTION : (10 points. NO PARTIAL CREDIT !!)

Find the equation of the line passing through the point $(2, 1)$ which cuts off the least area from the first quadrant.