

Do the problems in order in your bluebook. Show your work.

1. Find  $\int \frac{6x + 9}{x^2(x + 3)} dx$ .

2. Determine whether or not  $\int_{\pi^3}^{+\infty} \frac{x^5 + 3}{x^6 + 7x^2 + 401} dx$  converges.  
(Work out any integral you use in your comparison.)

3. Find  $\int_1^{e^e} \frac{\ln(x^3)}{x} dx$ .

4. Using integrals and Riemann sums, find  $\lim_{n \rightarrow +\infty} \left( \sum_{i=1}^n \frac{i^2}{n^3} \right)$

5. Find  $\int_0^{+\infty} x e^{-x} dx$ .

6. Find the area between the curves  $y = 4x$  and  $y = x^3$ .

7. Find  $\int_1^{+\infty} \frac{1}{x^2 + 1} dx$ .

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1. Find the volume of the solid obtained by rotating about the  $x$ -axis the region bounded by the curves  $y = x + e^x$ , the  $x$ -axis,  $x = 0$  and  $x = 1$ .
2. Set up (but don't evaluate) an integral that gives the surface area obtained by rotating the curve  $y = \tan(x)$  from  $x = 0$  to  $x = \pi$  about the  $x$ -axis.
3. Find the average value of  $y = \ln^2(x)/x$  over the interval  $[e, e^2]$ .
4. A heavy rope, 50 ft long, weighs 0.5 lb/ft and hangs over the edge of a tall building (taller than the length of the rope). How much work is done in pulling the rope to the top of the building ?
5. Find the centroid of the quarter unit-circle  $y = \sqrt{1 - x^2}$  for  $0 \leq x \leq 1$ . (You may use symmetry and the formula for the area of a circle.)
6. Find the mean of the probability density given by  $f(x) = \frac{1}{2\sqrt{2\pi}}e^{-x^2}$  for  $0 \leq x < +\infty$ .
7. Solve the differential equation  $y' = \frac{e^y}{1 + x^2}$
8. A tank contains 10 lbs of salt dissolved in 100 gallons of water. A fluid containing a salt solution with concentration of  $\frac{2}{10}$  lbs per gallon enters the tank at a rate of 3 gallons per minute. A well-stirred mixture leaves the tank at the same rate. Set up (but do not solve) an initial value problem that describes the amount of salt in the tank as a function of time.

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1. Define a sequence by  $a_1 = a_0 = 1$  and  $a_{n+1} = 2a_n + 3a_{n-1}$  for  $n \geq 1$ . It is easy to see that it diverges to infinity (don't bother showing this). However, the sequence of ratios of successive terms  $r_n = \frac{a_{n+1}}{a_n}$  converges. Find its limit.

2. Determine whether or not the series  $\sum_{n=1}^{+\infty} \frac{1}{1+n^2}$  converges

3. Find the radius of convergence of the power series  $\sum_{n=1}^{+\infty} \frac{2^n x^{2n}}{n}$ .

4. Find the Taylor series for  $y = \sin x^2$  and use the first three nonzero terms to approximate the integral  $\int_0^1 \sin x^2 dx$ .

5. Use power series to find the Taylor polynomial  $P_2$  of the solution to the initial value problem  $y' + 2y = x + 3x^2$  with  $y(0) = 2$ .

6. The planes  $x - y + 3z = 0$  and  $3x - 2y = 0$  both pass through the origin. Find the equation of the line of their intersection.

7. Find the equation of the plane that contains the points  $(0, 0, 1)$ ,  $(2, 1, 1)$  and  $(0, 2, 3)$ .

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1. Find the following:

(a)  $\int \frac{dt}{t^2 - 1}$

(b)  $\int x^3 e^{x^2} dx$

2. Determine whether or not the following converge (justify your argument):

(a)  $\int_3^{+\infty} \frac{x(1 + e^{-x})}{x^2 + 2\pi} dx$

(b)  $\int_5^{+\infty} \left( \frac{x^2 + \left( \sqrt{8 + \sin^2(x^6 + 1)} \right)}{x^4 + 1} \right) dx$

3. Find the area between the curves  $y = 2x^2 + 4x + 1$  and  $y = x^3 + x + 1$ .

4. Find the volume obtained by rotating about the  $x$ -axis the region bounded by  $y = e^x$ , the  $x$ -axis,  $x = 1$  and  $x = 2$ .

5. Find the average value of  $y = \tan^3(x) \sec^2(x)$  over the interval  $[0, \pi/4]$ . Draw a graph that shows the geometric significance of the average value.

6. A cylindrical water tank has height 20 feet and radius 6 feet. If the tank is half full of water, find the work required to pump all the water over the top rim. (Note: a cubic foot of water weighs 62.4 pounds.)

7. Solve the differential equation  $y' - x^2 y^2 = x^2$

8. Determine whether or not the series  $\sum_{n=1}^{+\infty} \frac{n}{e^{n^2}}$  converges

9. Find the MacLaurin series for  $y = \frac{3x^3}{7-x}$ . For what  $x$  does it converge ?

10. Find the equation of the plane that contains the points  $(1, -1, 2)$ ,  $(1, 0, 1)$  and  $(0, 2, 2)$ .