Math 126 Carter Final Fall 2003

Rules and Instructions. Indicate all work and answers in your blue book. Do not write on this test sheet. Write your name on only the outside of your blue book. Good luck.

1. Indicate the details of your computations for the following definite, indefinite, and improper integrals *(10 points each)*.

   (a) $\int \cos^2(x)\ dx$

   (b) $\int_1^3 x^5\ \ln(x)\ dx$

   (c) $\int \frac{1}{x^2-2x}\ dx$

   (d) $\int_1^\infty \frac{dx}{x^{1.3}}$

2. State the following *(10 points)*:

   (a) the definition of the definite integral of a real valued function that is defined on a closed inteval $[a, b]$;

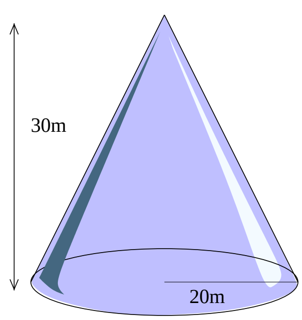
   (b) the fundamental theorem of calculus.

3. *(10 points)* Compute the area of the planar region that is bounded between $y = \sqrt{x}$, and $y = x^3$ for $x \in [0, 1]$.

4. *(10 points)* Use cylindrical shells to compute the volume of the solid obtained by revolving the portion of the graph of e^{-x^2} that is between $x = 0$ and $x = 1$ around the y-axis.

5. *(10 points)* Consider the cone illustrated. What is the volume of the cone?

   - First compute the volumes of cone by using the formula $V = \frac{\pi}{3}r^2h$.
   - Check your result by expressing the volume as an integral.



6. *(10 points)* Find the first 4 terms in the MacLaurin series expansion for the function $f(x) = \tan(x)$.

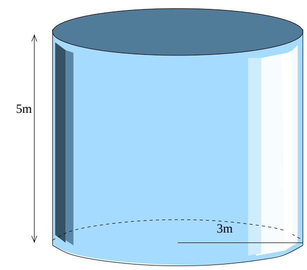
7. *(10 points)* Consider the differential equation:

$$y'' - 5y' + 6y = 0$$

   (a) Show that for any values of the constants A and B the function $y = Ae^{3x} + Be^{2x}$ satisfies the equation.

   (b) Determine the constants A and B above if $y(0) = 1; y'(0) = 2$.

8. *(10 points)* The cylindrical tank indicated is full of water (density 1000 Kilograms per cubic meters). Compute the work that it takes to empty the tank by pumping the water to the top of the tank.



9. *(10 points)* A bond that will pay \$50,000 in 6 years is sold at a cost of \$ 37,000. If you can earn 4.5% continuously compounded interest in an alternative investment, should you use the alternative investment or buy the bond?

10. *(10 points)* A sequence is given recursively by

$$a_1 = 1,$$

$$a_n = 1 + \frac{1}{1 + a_{n-1}}.$$

Give a list of the first 5 terms of the sequence. Assume that $\lim_{n\to\infty} a_n = L$ exists, to show that

$$L = 1 + \frac{1}{1 + L},$$

and use this fact to compute L.

11. *(10 points)* Use the power series expression for $f(x) = \frac{1}{1+x}$ to obtain a series expansion for $g(x) = x \ln(1 + x)$ that is valid at least in the interval $x \in (-1, 1)$. Hint: $\frac{d}{dx} \ln(1 + x) = \frac{1}{1+x}$, but that is not enough.

12. *(10 points)* Let $\vec{a}, \vec{b}$ denote vectors in 3-dimensional space ($\mathbb{R}^3$). Show that $||\vec{a} \times \vec{b}|| = ||\vec{a}||||\vec{b}|| \sin(\theta)$ where $||(a_1, a_2, a_3)|| = \sqrt{a_1^2 + a_2^2 + a_3^2}$ denotes the length of a vector and θ denotes the angle between the vectors.