

Math 126 Carter Test 3 Fall 2003

1. (10 points) Consider the differential equation:

$$y'' - 5y' + 6y = 0$$

- (a) Show that for any values of the constants A and B the function $y = Ae^x + Be^{2x}$ satisfies the equation.
- (b) Determine the constants A and B above if $y(0) = 0; y'(0) = 6$.
2. (20 points) A sample of *unobtainium 734* has a half-life of 30 years. If an initial sample of 45 grams is present, give an expression for the number of grams after t years. How many grams remain after 287 years? How long will it take until there are only 3 grams left?
3. (10 points) A sequence is given recursively by

$$a_1 = 1,$$

$$a_n = 1 + \frac{1}{1 + a_{n-1}}.$$

Give a list of the first 5 terms of the sequence. Assume that $\lim_{n \rightarrow \infty} a_n = L$ exists, to show that

$$L = 1 + \frac{1}{1 + L},$$

and use this fact to show that $L = \sqrt{5}$.

4. (10 points) Indicate a clear proof that

$$\lim_{n \rightarrow \infty} \frac{5^n}{n!} = 0.$$

5. Sum the series (10 points each)

(a)

$$\sum_{n=0}^{\infty} \left(\frac{1}{10}\right)^n$$

(b)

$$\sum_{n=1}^{\infty} \frac{1}{n(n-2)}$$

6. (10 points) Determine the interval of convergence of the series

$$\sum_{n=2}^{\infty} n(n-1)x^{n-2}$$

7. (20 points) Derive the Taylor (MacLaurin) series expansion for $\cos(x)$ by expanding around $a = 0$.