Fall 00 **Math 237 Exam 1 Review** Prof. Brick

1. Describe geometrically the set of vectors $v \in R^3$ with $(1,1,1) \cdot v \geq \frac{3}{2}|v|$.

2. Prove that $(1,3,2) + t(-4,0,12)$ and $(2,3,-1) + t(1,0,-3)$ describe the same line.

3. Solve the system of equations: $\left\{ \begin{array}{r} x + 4y + 2z = -2 \\ 6y + 7z = 1 \\ 2x + 9y = 5 \end{array} \right\}$ Describe things geometrically.

4. Given 3 equations in 3 unknowns, describe what the solutions can be geometrically ?

5. Find the nullspace of $\begin{pmatrix} 1 & 1 & 2 & 0 \\ 3 & 4 & 16 & 1 \\ 1 & 7 & 1 & 5 \end{pmatrix}$

6. Suppose $ad - bc = 0$. Prove that the columns of $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ are linearly dependent.

7. If A is a 5×7 matrix, what are the possibilities for the dimension of the nullspace.

8. Prove that the intersection of two subspaces is a subspace. What about unions ?

9. Prove that $A = \{(x,y,z) \in R^3 \mid x + 3y - 4z = 0\}$ is a subspace. Find a basis.

10. Determine if $\{(2,5,7,0), (3,2,-4,8), (4,1,0,1)\}$ are linearly independent.

11. Write (a,b) as a linear combination of $(4,3)$ and $(3,2)$.

12. Find a basis for the subspace of $M(2,2)$ consisting of the upper triangular matrices.

13. If $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent, then is $\{\vec{v}_1 + \vec{v}_3, \vec{v}_2 + 4\vec{v}_1, \vec{v}_3\}$?

14. Suppose $\vec{v}$ is in the span of $\{\vec{u}_1, \ldots, \vec{u}_n\}$. Can $\{\vec{v}, \vec{u}_1, \ldots, \vec{u}_n\}$ be linearly independent.

15. Suppose $\vec{u}, \vec{v}$ are both non-zero. Geometrically, how can $\vec{u}, \vec{v}$ be linearly dependent ?

16. Let $A = \begin{pmatrix} 1 & 3 & 2 & 2 \\ 4 & 0 & 3 & 1 \\ 0 & 1 & 1 & 0 \\ 5 & 2 & 4 & 3 \end{pmatrix}$ Find a maximal set of independent columns. Express the other columns as linear combinations of these columns.

17. Determine if $\begin{pmatrix} 5 & 3 \\ 2 & 1 \end{pmatrix}$ is a linear combination of $\begin{pmatrix} 4 & 3 \\ 4 & 2 \end{pmatrix}$ and $\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$ in $M(2,2)$

18. Review all the quizzes and all the homework.

19. Review all the quizzes and all the homework again.

20. Go over everything else.

MA 237, Fall 00 **Linear Algebra I — Exam 1** Prof. Brick

Do the problems in order in your bluebook. Show your work.

1. Using a matrix approach, solve the system of equations: $\left\{ \begin{aligned} x_1 + 4x_2 + 3x_3 + x_4 &= -2 \\ 2x_2 + 4x_3 &= 1 \\ 2x_2 + 5x_3 &= 5 \end{aligned} \right\}$

2. Find spanning vectors for the nullspace of the matrix $\begin{pmatrix} 1 & 0 & 0 & 9 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

3. Using matrices, decide whether $\Big\{(1,0,0,0), (0,1,1,0), (1,0,1,0), (2,1,3,0)\Big\}$ is linearly independent or linearly dependent.

4. Prove that $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, and $C = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ is a basis for the subspace of $M(2,2)$ consisting of upper triangular matrices.

5. Write (a, b) as a linear combination of $(4, 3)$ and $(3, 2)$.

6. Let P_2 be the vector space of polynomials of degree ≤ 2. Define

$$S = \{p(x) \in P_2 \,|\, p(0) = 0\}.$$

Prove that S is a subspace of P_2.

7. Suppose $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent. Prove that $\{\vec{v}_1 + \vec{v}_3, \vec{v}_1 + 4\vec{v}_2, 2\vec{v}_2 - \vec{v}_3\}$ is also linearly independent. Use matrices in your argument.

Fall 00 **Math 237 Exam 2 Review** Prof. Brick

1. Suppose A is a (72×43)–matrix with dim(row space) $= 25$. Find the dim(null(A)).

2. Suppose $A = \begin{pmatrix} a & b & c & d & e \\ f & g & h & i & j \\ k & l & m & n & p \end{pmatrix}$ row reduces to $\begin{pmatrix} 1 & 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

(a) Find a basis for the row space of A
(b) Find a basis for the column space of A
(c) Find a basis for the kernel of A.

3. Find the matrix of the linear map written with variables as $f(x, y, z) = (\pi y - 5z, 2x - y)$.

4. If $\begin{pmatrix} 4 & 0 & 3 & 5 \\ \pi & 17 & 0 & \sqrt{3} \\ 2 & 1 & 3 & 0 \end{pmatrix}$ is the matrix of a linear map f, find f using variables.

5. Find the eigenvalues and eigenspaces of $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$

6. Find a basis for the image of the linear map with matrix $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 4 & 1 & 7 \\ 0 & 1 & 0 & 0 \end{pmatrix}$

7. Find all matrices that commute with $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

8. Suppose A and B are $(n \times n)$-matrices with $AB = I_{n \times n}$, the $(n \times n)$-identity matrix. Find the image of B.

9. Find the inverse, if it exists, of the matrix $\begin{pmatrix} 0 & 3 & 2 \\ 1 & 2 & 5 \\ 0 & 4 & 1 \end{pmatrix}$

10. Suppose A and B are invertible $(n \times n)$-matrices. Prove that AB is invertible. How about $A + B$?

11. Find the det(A) where $A = \begin{pmatrix} 5 & 0 & 0 & 0 \\ 2 & 9 & 1 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 8 & 1 & 0 \end{pmatrix}$

12. Suppose A and A^{-1} are matrices with integer entries. What could det(A) equal ?

13. Suppose f is linear. Prove that f is one-to-one iff dim(null(f))=0.

14. Suppose $A^2 = A$. What are the possible eigenvalues of A ?

15. Find the eigenvalues and eigenspaces of $\begin{pmatrix} 1 & 0 \\ 9 & 0 \end{pmatrix}$

16. Review everything else.

Fall 00 Prof. Brick

Math 237 Final Exam Review

1. Go over all the quizzes, all the homework, and the previous review sheets.

2. Show that $u = (3, 1, 2)$, $v = (2, 0, 1)$ and $w = (5, -2, 7)$ form a basis for R^3. Find the transition matrices between this basis and the standard basis.

3. Suppose $f : R^3 \to R^3$ is the linear map given by $f(u) = 2u - 3v$, $f(v) = 5w$ and $f(w) = 3u + w - u$. Find the matrix of f with respect to the standard basis.

4. Find an orthonormal basis for the subspace of R^5 spanned by $(3, 6, 7, -2, 1)$, $(4, 1, 0, 3, 4)$, and $(1, 2, 0, 2, 9)$.

5. Prove that the inverse of an orthogonal matrix is orthogonal.

6. Go over all the quizzes, all the homework, and the previous review sheets.

7. Find the eigenvalues and eigenvectors of $\begin{pmatrix} 0 & 3 & -3 \\ 2 & 2 & -2 \\ -4 & -1 & 1 \end{pmatrix}$

8. Express the variety $18x^2 + 5y^2 + 12z^2 - -12xy - 12yz = 7$ in standard form. Give the new orthonormal basis.

9. Let a, b, c be numbers with $a + c > 0$. Prove that $ax^2 + 2bxy + cy^2 = 1$ defines an ellipse if and only if $b^2 < ac$.

10. Suppose A is a diagonalizable $n \times n$-matrix which has only 2 and 4 as eigenvalues. Prove that $A^2 - 6A + 8I_{n\times n} = 0_{n\times n}$

11. Let $S = \{v_1, v_2, \ldots, v_9\}$ be a set of vectors in R^{45}. Denote by $S^\perp$ the set of $w \in R^{45}$ with $w \cdot v_i = 0$ for $i = 1, 2, \ldots, 9$. Prove that $S^\perp$ is a subspace.

12. Go over all the quizzes, all the homework, and the previous review sheets.

13. Review everything else.

MA 237, Fall 00 **Linear Algebra I — Final Exam** Prof. Brick

Do the problems in order in your bluebook. Use methods from class only. Show your work. Justify your answers. You may use a calculator only for numerical calculations.

1. State the definition of "linear map" without refering to matrices.

2. Explain how you would use row reductions to find the inverse (but don't actually find an inverse) of a 7×7-matrix. Describe what would happen if you started with a non-invertible matrix.

3. Suppose $A = \begin{pmatrix} a & b & c & d & e \\ f & g & h & i & j \\ k & l & m & n & p \end{pmatrix}$ row reduces to $\begin{pmatrix} 1 & 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

Find bases for the row space, the column space, and the kernel of A.

4. Let $S = \{v_1, v_2, \ldots, v_9\}$ be a set of vectors in R^{45}. Denote by $S^\perp$ the set of $w \in R^{45}$ with $w \cdot v_i = 0$ for $i = 1, 2, \ldots, 9$. Prove that $S^\perp$ is a subspace.

5. Suppose $\vec{a} = (a_1, a_2, a_3)$, $\vec{b} = (b_1, b_2, b_3)$ and $\vec{c} = (c_1, c_2, c_3)$ form a basis for R^3. Let $f : R^3 \to R^3$ be the linear map defined by $f(\vec{a}) = \pi \cdot \vec{a} - \sqrt{2} \cdot \vec{c}$, $f(\vec{b}) = 12 \cdot \vec{a} + \frac{1}{2} \cdot \vec{b} + 9 \cdot \vec{c}$, and $f(\vec{c}) = 666 \cdot \vec{a}$. Find a matrix expression for the matrix of f with respect to the standard basis of R^3.

6. Let $\vec{u} = (1, 2, -2, 4)$, $\vec{v} = (-4, 2, 2, 1)$, and $\vec{w} = (0, 0, 1, 3)$. Show that $\vec{u}$ and $\vec{v}$ are orthogonal. Then find an orthogonal basis for the subspace of R^4 spanned by $\vec{u}, \vec{v}, \vec{w}$.

7. Use matrices to solve the system of equations: $\left\{ \begin{array}{c} x + 2y + z = 5 \\ x + y + 2z = 3 \end{array} \right\}$. Describe the set of solutions geometrically.

8. Find the eigenvalues and bases of the corresponding eigenspaces of $\begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}$.

9. Suppose A is a (95×61)–matrix with dim(row space) $= 43$. Find the dim(null(A)).

10. Express the variety $x^2 + y^2 + 6xy = 7$ in standard form. Give the orthonormal basis that yields this standard form. Use matrices in your approach.

11. Suppose $\vec{a} = (a_1, a_2, a_3)$, $\vec{b} = (b_1, b_2, b_3)$ and $\vec{c} = (c_1, c_2, c_3)$ forms an orthogonal basis for R^3. Consider the matrix A whose columns are $\vec{a}$, $\vec{b}$, and $\vec{c}$. Find A^{-1}.

12. Suppose $\vec{a} = (a_1, a_2, a_3)$, $\vec{b} = (b_1, b_2, b_3)$ and $\vec{c} = (c_1, c_2, c_3)$ form a basis for R^3. Find a matrix expression for a 3×3-matrix with eigenvalues 7 and 42 where the eigenspace corresponding to 7 has basis $\{\vec{a}, \vec{b}\}$, and that of 42 has basis $\{\vec{c}\}$

13. Suppose A is a diagonalizable $n \times n$ matrix whose only eigenvalues are 1 and -1. Prove that $A^2 = I_{n\times n}$ (the $n \times n$ identity matrix).