Spring '01 **Math 238 Exam 1 Review** Prof. Brick

1. A tank with a capacity of 1000 gal initially contains 100 gal of water with 50 lb of salt dissolved in it. Water containing 2 lb of salt per gal flows into the top of the tank at a rate of 4 gal/min. A well-stirred solution flows out of the bottom of the tank, but at a rate of 3 gal/min. How much salt is in the tank before it starts to overflow ?

2. To celebrate getting an "A" in Math 238, you go out drinking. But you very foolishly decide to drive (and you should be ashamed of yourself). You get arrested at 10pm. At 11pm, your blood alcohol is measured and found to be 0.08%. An hour later, it is measured again and is found to be 0.05%. If the legal limit for alcohol content in the blood is 0.10%, were you legally drunk when you were arrested ?

3. The half-life of $C_{14}$ is 5568 years. How old is a fossil that contains 10% of the $C_{14}$ found in living things ?

4. Solve the following DE's:

(a) $\dfrac{dy}{dt} + 2ty = t$ (b) $\dfrac{dy}{dt} + \dfrac{1}{t}y = \sin(t), \quad t > 0$

5. Solve the following IVP's:

(a) $t\dfrac{dy}{dt} - 3ty = te^{4t}, \quad y(0) = 1$ (b) $\dfrac{dy}{dt} - y = t, \quad$ y(0)=2

6. Solve the following equations, obtaining an explicit solution if possible:

(a) $(y^2(1+x^2))\dfrac{dy}{dx} = \arctan(x)$ (b) $\dfrac{dy}{dx} = \dfrac{xe^x - 1}{ye^x + e^{(y+x)}}$

7. Without solving them, use a slope field approach to sketch the solutions to the following differential equation. Indicate those solutions that are equilibriums. You should be able to do this without your calculator.

(a) $\dfrac{dy}{dt} = -y(y-5)$ (b) $\dfrac{dy}{dt} = -(y-1)^2(y-4)$

8. Using a two compartment model, give a linear cascade system of DE's that describes the effect of taking a cold pill if the observed data shows an absorption half-life of one hour and a clearance half-life (after maximum absorption) of eight hours. Solve the system of equations obtained.

9. Find the first three Picard iterates for the IVP $y' = x - y^2$ with $y(0) = 2$.

Spring '01 **Math 238 Exam 1** Prof. Brick

**Do the problems in order in your bluebook. Show your work.**

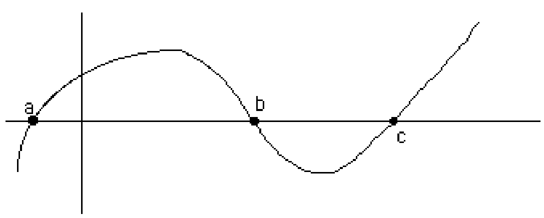
1. A tank with a capacity of 1000 gal initially contains 100 gal of water with 50 lb of salt dissolved in it. Water containing 2 lb of salt per gal flows into the top of the tank at a rate of 4 gal/min. A well-stirred solution flows out of the bottom of the tank, at a rate of 4 gal/min. Set up, but do not solve, an IVP for the amount of salt in the tank.

2. To celebrate getting an "A" in Math 238, you go out drinking. But you very foolishly decide to drive (and you should be ashamed of yourself). You get arrested at 10pm. At 11pm, your blood alcohol is measured and found to be 0.09%. An hour later, it is measured again and is found to be 0.07%. If the legal limit for alcohol content in the blood is 0.10%, were you legally drunk when you were arrested ? Use and solve a DE in your answer.

3. Solve $\dfrac{dy}{dt} + 2ty = t, \quad y(0) = 1$

4. Solve $t\dfrac{dy}{dt} - 3ty = te^{4t}$

5. Solve $\dfrac{dy}{dx} = \dfrac{x}{y}e^{x}, \quad y(0) = -2$




6. Sketch the solutions to $\dfrac{dy}{dt} = f(y)$, where the graph of $f$ is pictured above. Indicate those that are equilibriums. Note: assume $t \geq 0$ and $y$ may take on negative values.

7. Using a two compartment model, give a linear cascade system of DE's that describes the effect of taking a cold pill if the observed data shows an absorption half-life of two hours and a clearance half-life after maximum absorption of nine hours. Set up but do not solve the system of equations obtained.

8. Find the first three Picard iterates for the IVP $y' = xy$ with $y(0) = 2$.

Spring '01 **Math 238 Exam 2 Review** Prof. Brick

1. Solve the initial value problem $y'' - y' - 2y = 0$, $y(0) = \alpha$ and $y'(0) = 2$. For what value of $\alpha$ does the solution approach zero as $t \to +\infty$ ?

2. Solve the inital value problem $y'' - 2y' + 5y = 0$, $y(\pi/2) = 0$, and $y'(\pi/2) = 2$.

3. Solve the inital value problem $y'' + 4y' + 4y = 0$, $y(-1) = 2$, and $y'(-1) = 1$.

4. Solve $\Big(3x^2\sin(y) + e^x\Big)dx - \Big(\sin(y) - x^3\cos(y)\Big)dy = 0$

5. Solve $(2x^2 + xy)y' = y^2 - 2xy$

6. Solve $y'' - 2y' - 3y = 3e^{-11t}$

7. Solve $y'' + y = \sin(2t)$ with $y(0) = 1$, $y'(0) = 2$

8. Suppose a mass weighing 4lb stretches a spring 3 in. Assume the damping constant is 2 lb-sec/ft and that there is an external force of $2\cos(3t)$ lb. The mass is pushed upward 1 in and then let go with no initial velocity. Set up an initial value problem describing the motion of the spring.

9. Suppose a mass weighing 4lb stretches a spring 3 in. Assume the damping constant is $\alpha$ lb-sec/ft and that there is no external force. The mass is pushed upward 1 in and then let go with no initial velocity. For what value of $\alpha$ is the spring overdamped ?

10. Find a second order linear differential equation with constant coefficients for which $y_1 = t^2 + 2^t$ and $y_2 = t^2 - 3^t$ are solutions.

11. Consider $x" + x' - x^2 = 0$. Find a formula for $\dfrac{dy}{dx}$ that one would use to plot the phase plane. Find the equation of the nullcline. Describe the regions of the phase plane for which the orbits have positive slope.

12. Review everything else.

Spring '01 **Math 238 Exam 2** Prof. Brick

**Do the problems in order in your bluebook. Show your work.**

1. Solve the initial value problem $y'' - 6y' - y = 0$, $y(0) = \alpha$ and $y'(0) = 5$. For what value of $\alpha$ does the solution approach zero as $t \to +\infty$ ?

2. Solve $y'' + y' = \sin(3t)$.

3. Suppose a mass weighing 32 lb stretches a giant spring 8 feet. Assume the damping constant is $\alpha$ lb-sec/ft and that there is no external force. The mass is pulled downwards 6 feet and then let go with no initial velocity. Set up an IVP, but do not solve it. Without completely solving the IVP, determine what value of $\alpha$ causes the spring to be crticially damped or overdamped.

4. Consider $x" + x' - x^2 = 0$. Using $y = x'$ find a formula for $\frac{dy}{dx}$ that one would use to plot the orbits in the phase plane. Find the equation of the nullcline (where the orbits have horizontal slope). Describe the regions of the phase plane for which the orbits have positive slope.

5. Find a second order linear differential equation with constant coefficients (not necessarily homogeneous) for which $y_1 = t^2 + c_1 2^t + c_2$ is a solution for all constants $c_1$ and $c_2$.

6. Solve $(\sin(y) - x^3 \cos(y))\frac{dy}{dx} = 3x^2 \sin(y) + e^x$

7. Solve the inital value problem $y'' - 2y' + 5y = 0$, $y(0) = 3$, and $y'(0) = 1$.

Spring '01 **Math 238 Final Review** Prof. Brick

1. Go over the review sheets from the previous exams.

2. Use the method of undetermined coefficients to solve $y'' - 2y' = 5e^{2t}$.

3. Use the method of undetermined coefficients to solve $y'' + 6y' + 9y = \cos(3t)$.

4. Use the method of undetermined coefficients to solve $y'' + 2y = t^2 + 4t + 3$.

5. Find a second order linear differential equation with constant coefficients for which $y_1 = c_1 + c_2 2^t + \tan(t)$ are solutions for any constants $c_1$ and $c_2$.

6. Suppose $f(t) \geq \frac{e}{\sqrt{\pi}}$ for all $t$. What can you say about solutions to $x'' + f(t) \cdot x = 0$ ?

7. Describe the nature of the critical point $(0, 0)$ of the system $x' = y$ and $y' = -5x - 4y$.

8. Describe the nature of the critical point $(0, 0)$ of the system $x' = -2x$ and $y' = -y$.

9. Describe the nature of the critical point $(0, 0)$ of the system $x' = y + 3x$ and $y' = x - 4y$.

10. Without using a table of transforms, and without doing any integration by parts, find the Laplace transform of $t^3$.

11. Find the inverse Laplace transform of $\dfrac{s^2 + 4s + 7}{(s + 4)(s - 6)(s^2 + 1)}$

12. Using the Laplace transform, change the IVP $y'' + 5y' - 6y = 8\cos(2t)$ with $y(0) = 2$ and $y'(0) = 8$ into an algebraic equation of the form $L(y) =$ "some expression in $s$".

13. Using integrals (and no tables), find the Laplace transform of the step function

$$f(t) = \begin{cases} 0, & \text{if } x \leq 3 \\ 1 & \text{otherwise} \end{cases}$$

14. Review everything else.

Math 238 **Differential Equations – Final Exam** Prof. Brick
section 2 Spring '01

**Do the problems in order in your bluebook. Show your work.**

1. For what value of $\alpha$ does the solution to $y'' - y' - 6y = 0$, $y(0) = \alpha$ and $y'(0) = 6$. approach zero as $t \to +\infty$ ?

2. A tank with a capacity of 500 gal initially contains 200 gal of water with 50 lb of salt dissolved in it. Water containing 3 lb of salt per gal flows into the top of the tank at a rate of 5 gal/min. A well-stirred solution flows out of the bottom of the tank, but at a rate of 3 gal/min. How much salt is in the tank before it starts to overflow ?

3. Suppose a mass weighing 16 lb stretches a giant spring 6 feet. Assume the damping constant is $\alpha$ lb-sec/ft and that there is no external force. The mass is pulled downwards 6 feet and then let go with no initial velocity. Set up an IVP, but do not solve it. Without completely solving the IVP, determine what value of $\alpha$ causes the spring to be either crticially damped or overdamped.

4. Solve $\dfrac{dy}{dt} = \dfrac{te^{3t} - y}{t}, \quad t > 0$

5. Use a slope field approach to sketch the solutions to $\dfrac{dy}{dt} = (5 - y)(y - 2)$ Indicate those solutions that are equilibriums.

6. Using integrals (and no tables), find the Laplace transform of the step function

$$f(t) = \begin{cases} 0, & \text{if } x \leq 5 \\ 1 & \text{otherwise} \end{cases}$$

7. Using a two compartment model, give a linear cascade system of DE's that describes the effect of taking a cold pill if the observed data shows an absorption half-life of three hours and a clearance half-life (after maximum absorption) of five hours. Do not solve the system of equations obtained.

8. Describe the nature of the critical point $(0, 0)$ of the system $x' = y$ and $y' = -5x - 4y$.

9. Without using a table of transforms, and without doing any integration by parts, find the Laplace transform of $t^2$.

10. Solve $(\sin(y) - x^3 \cos(y))\dfrac{dy}{dx} = 3x^2 \sin(y) + e^x$

11. Find the inverse Laplace transform of $\dfrac{2s + 5}{(s + 4)(s - 6)}$

12. Using the Laplace transform, change the IVP $y'' + 2y' - 7y = 7\cos(5t)$ with $y(0) = 2$ and $y'(0) = 8$ into an algebraic equation of the form $L(y) =$ "some expression in $s$".