Fall '02 **Math 267 Exam 1 Review** Prof. Brick

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1. Suppose $p =$"I love Math", $q =$"I eat hot dogs", and $r =$"puppies like me." Express in English the meaning of $(p \wedge (\neg q)) \rightarrow r$.

2. Determine whether or not the propositions $p \rightarrow q$ and $\neg((\neg q) \wedge p)$ are logically equivalent.

3. Using a first order language with universe being the set of integers, with the constant 1 and the function multiplication "$\cdot$" — and only those extras — write a mathematical sentence that says that the square root of 2 is irrational.

4. Find $\sum_{i=1}^{5} \left(\sum_{j=3}^{i} (5 - 2j) \right)$

5. Let $P(x, y)$ denote the statement "x likes y" where the domain of discourse for x and y is the set of people in our class. Express clearly and precisely in proper non-mathematical English what $\forall y(\exists x P(x, y))$ means.

6. Consider the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$. If bit strings are used to describe the subsets of U, what operation on the bit strings corresponds to intersection of sets.

7. Explain briefly what is meant by "P" and "NP" in "P=NP". Besides the million dollars you would win, what would be the significance of a proof of "P=NP" ?

8. Give a recursive formula that computes the sum of the first n odd integers.

9. Prove that $f_1^2 + f_2^2 + \ldots + f_n^2 = f_n \cdot f_{n+1}$ where f_k is the kth Fibonacci number.

10. Define the set of strings B recursively by saying it contains the empty string λ, is closed under concatenation and if $x \in B$ then $(x) \in B$. Define a function N on B by $N(\lambda) = 0$, $N(() = +1$, $N()) = -1$ and $N(xy) = N(x) + N(y)$. Prove that $w \in B$ iff $N(w) = 0$ and N is nonnegative on all prefixes (also known as initial segments) of w.

11. Prove that 5 divides $n^5 - n$ for any nonnegative integer n.

12. Explain the difference between an indirect proof and a proof by contradiction.

13. What is the least power function that is the same O-type as $f(n) = 3n^4 - 2n^2 + 7n + 11$.

14. Find the gcd of 120 and 32. Show all your steps.

15. Prove that $\sqrt{2}$ is irrational, that there are an infinite number of primes, and that the reals are uncountable.

16. Give a recursive definition for the reversal of a string (where that means the string written backwards).

17. Use induction and recursion to explain Pascal's triangle and its connection to the binomial expansion.

18. Give a recursive algorithm to check whether a string is a palindrome

19. Describe the set of bit strings which are palindromes using recursion on sets.

20. Review all the homework, all the quizzes, your notes from class, and everything else.