

Do the problems in order in your bluebook. Show all your work and explain your answers.

1. Consider the set of ternary strings . Define a relation R by wRu if w and u agree on the first n characters where n is at least as large as the minimum of half of the lengths of w and u . Prove or disprove: R is an equivalence relation.

2. Give the recurrence relation for the number of bit strings of length n that do not contain two consecutive ones. Explain your reasoning.

3. Consider the set of subsets of the set $\{a, b\}$ and the partial ordering given by inclusion. Draw the Hasse diagram.

4. Define what is meant by “same O -type” for functions. Explain how and why it might be a useful concept in computer science.

5. Give a combinatorial proof that $2^n = \sum_{k=0}^n C(n, k)$

6. A student needs to choose ten electives from Mathematics, Statistics, Physics, and Chemistry. Ignoring prerequisites and other complications, how many different selections – in terms of how many in each of the various disciplines – are possible ?

7. For what values of n is the K_n Eulerian ? Prove it.

8. Draw $K_{2,3}$ and find its incidence matrix.

9. Consider the recursive definition for the set of strings S over the alphabet consisting of two symbols (and) :

$\lambda \in S$ (where λ is the empty string),

if $w \in S$ then $(w) \in S$.

if $w \in S$ and $u \in S$ then $wu \in S$.

Give a non-recursive description of the set S .

10. Give a recursive definition for the reversal of a string.

11. State the generalized pigeonhole principle. Prove it by contradiction.

12. Describe what a deterministic finite state automaton is and what is meant by the language it accepts. Do not just describe an example.

13. Describe what a proof by induction is. Be succinct. Do not just describe an example.

14. Describe what $P = NP$ is and how a proof of it might impact today’s world.