Math 267 — Prof. Brick
section 1 — Spring 02

Discrete Math Final Exam

Do the problems in order in your bluebook. Show your work and justify your answers.

1. How many ternary strings of length 16 contain exactly 5 one's ?

2. A jar contains hundreds of marbles colored blue, green, red, and yellow. Assuming you are blindfolded, how many marbles must you take to be sure to get 5 of the same color ? Explain it using the pigeonhole principle.

3. The entries of Pascal's triangle are of the form $C(n, k)$. It is a theorem that the $(n+1)$-th row can be gotten from the n-th row. State how and give a combinatorial proof.

4. Suppose $X \subset Y$ are graphs. Prove that the chromatic number of X is less than or equal to the chromatic number of Y.

5. Define what a characteristic function is. State its domain and codomain. What operation on characteristic functions corresponds to intersections of sets ?

6. Prove that the relation on integers of "divides" is transitive. Prove that it is not an equivalence relation.

7. Find the recurrence relation with initial conditions for the number of ways of climbing n stairs, assuming you can take them one or two stairs at a time.

8. Consider the set of subsets of the set $\{1, 2, 3\}$ and the partial ordering given by inclusion. Draw the Hasse diagram.

9. Let $f : X \to Y$ be a function. Define a relation R on X by $x_1 R x_2$ iff $f(x_1) = f(x_2)$. Prove that R is an equivalence relation.

10. Define the terms "Eulerian circuit", and "Hamiltonian circuit" for graphs. What condition is equivalent to the existence of an Eulerian circuit ?

11. State Kuratowski's theorem about non-planar graphs.

12. Let $P(x, y)$ denote the statement "x likes y" where the domain of discourse for x and y is the set of people in our class. Express clearly and precisely in proper **non-mathematical** English what $\forall y(\exists x P(x, y))$ means. Make sure your answer is not ambiguous.