Spring '02 **Math 267 Final Exam Review** Prof. Brick

1. State and prove the two theorems about chromatic numbers from class. Use them to construct a graph having 505 vertices and yet with chromatic number 5.

2. How many bit strings of length 8 do not contain two consecutive 0's.

3. A jar contains marbles colored blue, green, red, and yellow. Assuming you are blindfolded, how many marbles must you take to be sure to get 7 of the same color ?

4. Explain what $P = NP$ is all about.

5. Give a combinatorial proof that $C(n, k) = C(n, n - k)$.

6. Give a combinatorial proof that $2^n = \sum_{k=0}^{n} C(n, k)$

7. An exam consists of 35 true-false questions. Exactly ten of the questions have "true" as the answer. How many answer keys are possible.

8. Prove that 5 divides $n^5 - n$ for all nonnegative integers n.

9. You draw 2 cards. Find the probability that the second one is a king given that the first one is a heart.

10. Find the probability of getting a full house in 5-card poker.

11. Find the recurrence relation for the number of ways of climbing n stairs, assuming you can take them one or two stairs at a time.

12. Prove that the relation on integers of "divides" is transitive.

13. Explain what big O-type means and what it could be used for in theoretical computer science. What are some of the practical limitations of the concept.

14. Consider the set of subsets of the set $\{1, 2, 3\}$ and the partial ordering given by inclusion. Draw the Hasse diagram.

15. Define "equivalence relation".

16. Prove that the sum of the squares of the first n Fibonacci numbers is equal to $f_n \cdot f_{n+1}$.

17. Let $P(x, y)$ denote the statement "x likes y" where the domain of discourse for x and y is the set of people in our class. Express clearly and precisely in proper non-mathematical English what $\forall y(\exists x P(x, y))$ means.

18. Define the set of strings B recursively by saying it contains the empty string λ, is closed under concatenation and if $x \in B$ then $(x) \in B$. Define a function N on B by $N(\lambda) = 0$, $N(() = +1$, $N()) = -1$ and $N(xy) = N(x) + N(y)$. Prove that $w \in B$ iff $N(w) = 0$ and N is nonnegative on all prefixes (also known as initial segments) of w.

19. Find the gcd of 120 and 32. Show all your steps.

20. What operation on characteristic functions corresponds to unions of sets.

21. Explain the difference between an indirect proof and a proof by contradiction.

22. Determine which of the graphs on pages 463-467 are Eulerian.

23. If possible, determine which of the graphs on pages 485-488 are Hamiltonian.

24. Explain Djikstra's algorithm

25. Do problems 1-23 on pages 508-510

26. Review all the homework, all the quizzes, the previous review sheets, the previous exams, and your notes from class. Then review everything else.