

Do the problems in order in your bluebook.

1. Define the following terms:

(a) the index of a subgroup

(b) homomorphism

2. Let A and B be groups. Suppose $f, g : A \rightarrow B$ are homomorphisms. Show that $E = \{x \in A \mid f(x) = g(x)\}$ is a subgroup of A .

3. List the left cosets of $\langle (1234) \rangle$ in S_4 .

4. Suppose G is a finite group of order p where p is a prime. Let $x \in G$ be any element different from the identity. Prove that $G = \langle x \rangle$.

5. Let $H = \{ \sigma \in S_9 \mid \sigma(5) = 5 \}$ Prove that $(5\ 6)H = \{ \tau \in S_9 \mid \tau(5) = 6 \}$.

6. Suppose $f : G \rightarrow H$ is a homomorphism. Show that $|f(x)|$ divides $|x|$.

7. Consider the group $G = Z_6 \oplus Z_4 / \langle (2, 2) \rangle$. Find all of its elements and their orders.

Do the problems in order in your bluebook.

1. Define the following terms:

(a) the index of a subgroup

(b) homomorphism

2. Let A and B be groups. Suppose $f, g : A \rightarrow B$ are homomorphisms. Show that $E = \{x \in A \mid f(x) = g(x)\}$ is a subgroup of A .

3. List the left cosets of $\langle (1234) \rangle$ in S_4 .

4. Suppose G is a finite group of order p where p is a prime. Let $x \in G$ be any element different from the identity. Prove that $G = \langle x \rangle$.

5. Let $H = \{ \sigma \in S_9 \mid \sigma(5) = 5 \}$ Prove that $(5\ 6)H = \{ \tau \in S_9 \mid \tau(5) = 6 \}$.

6. Suppose $f : G \rightarrow H$ is a homomorphism. Show that $|f(x)|$ divides $|x|$.

7. Consider the group $G = Z_6 \oplus Z_4 / \langle (2, 2) \rangle$. Find all of its elements and their orders.

Do the problems in order in your bluebook. Justify your answers.

1. Define the following terms:

(a) group (b) the order of an element (c) quotient group

2. Prove that in a group we have $(ab)^{-1} = b^{-1}a^{-1}$

3. Prove that $G = \{a + b\sqrt{2} \mid a, b \in \mathbb{Z}\}$ is a subring of the reals.

4. How many abelian groups of order 450 are there up to isomorphism? List them.

5. Give an example of a commutative ring which is not an integral domain.

6. Find the cyclic subgroup of S_9 generated by the element $(2\ 5\ 7)(5\ 1\ 4\ 3)$.

7. Suppose G is a group with $x^2 = e$ for all x . Prove that G is abelian.

8. Let A and B be groups. Suppose $f, g : A \rightarrow B$ are homomorphisms. Show that $E = \{x \in A \mid f(x) = g(x)\}$ is a subgroup of A .

9. Suppose G is a finite group of order p where p is a prime. Let $x \in G$ be any element different from the identity. Prove that $G = \langle x \rangle$.

10. List the left cosets of $\langle (1234) \rangle$ in S_4 .