Spring '00 **Math 502 Exam 1 – in class part** Prof. Brick

1. Define the term "the order of an element".

2. Find all of the subgroups of Z_{12}

3. Show that $\{2, 5\}$ generates Z_{10}, but neither $\{2\}$ nor $\{5\}$ do. Then draw the Cayley graph for Z_{10} using this generating set.

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4. Suppose G is a group with $x^2 = e$ for all x. Prove that G is abelian.

5. Prove that the intersection of two subgroups is itself a subgroup and show by example that the union of two subgroups need not be a subgroup.

6. Two elements x and y of a group G are said to be conjugate if there exists some element a so that $y = a \cdot x \cdot a^{-1}$. Prove that conjugate elements have the same order.

7. Let H be a subgroup of G. Define a relation $\equiv_H$ on G by saying that $a \equiv_H b$ iff $a \cdot b^{-1} \in H$. Prove that $\equiv_H$ is an equivalence relation on G.

Spring '00 **Math 502 Exam 2 – take home** Prof. Brick

– due Tuesday, April 18 –

Note: Class will not meet on Thursday, April 13.

1. List the left cosets of $\Big\langle(1\ 2\ 3\ 4)\Big\rangle$ in S_4.

2. Suppose H is a subgroup of G. Define $N(H) = \{x \in G \,|\, xHx^{-1} = H\}$ Prove that $N(H)$ is a subgroup of G containing H, in which H is normal.

3. Suppose A and B are subgroups of G and with B normal. Prove that AB is a subgroup.

4. List all the left cosets of $\big\langle(1\ 2)\big\rangle$ in S_3. Show that $\big\langle(1\ 2)\big\rangle$ is not normal.

5. Classify the group $G = Z_5 \oplus Z_{12}/\langle(2,2)\rangle$.

6. Let G be a group and H a normal subgroup with $[G : H] = 3$. Prove that $[G, G] \subset H$. (Hint: look at the natural map $G \to G/H$)

7. List, up to isomorphism, all abelian groups of order 1800.

8. Suppose $N \triangleleft G$ with $N \cap [G, G]$ trivial. Prove that $N \subset Z(G)$.

Spring '00 **Math 502 Final Exam** Prof. Brick

Do the problems in order in your bluebook. Show your work. Justify your answer.

1. Define the following terms:
(a) group (b) subgroup (c) the index of a subgroup
(d) integral domain (e) ring (f) field

2. Give an example of an infinite non-abelian group.

3. Give an example of an infinite non-commutative ring.

4. Explain why the group $Z_4 \times Z_4$ is not isomorphic to the group $Z_2 \times Z_8$.

5. Find the cyclic subgroup of S_9 generated by the element $(4\ 5\ 7)(5\ 2\ 4\ 3)$.

6. Suppose G is a group with $x^2 = e$ for all x. Prove that G is abelian.

7. Prove that $K = \{a + b\sqrt{3} \,|\, a, b \in Q\}$ is a subfield of the reals.

8. Give an example of a commutative ring which is not an integral domain. Explain why.

9. How many abelian groups of order 36 are there up to isomorphism ? List them.

10. True or False:
(a) An n degree polynomial over a ring can have at most n roots.
(b) Angles cannot be trisected using a compass and straightedge.
(c) Abstract Algebra is way cool.