Prof. S. Brick **Math 502**
Spring '00 **section 51**

Introduction to Abstract Algebra; Quiz 0

1. <u>Print</u> your name. What is your year in school and your major ?

2. Are you a teacher ? If so, where and at what level ?

3. What was the last math class you took ? When ?

4. What is abstract algebra ?

5. Which of the following is familiar to you:
(a) proof by induction **(b) prime numbers** **(c) matrices**

6. What other math classes do you plan to take ?

7. What grade do you honestly expect to get from this class ? Why ?

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Abstract Algebra; Quiz 1

0. Print your name: ______________________________

1. Define R on the set of integers by nRm iff $3|(n-m)$. Prove R is an equivalence relation.

2. Determine whether subtraction is an associative binary operation on the set of integers.

3. Define the term "group".

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Abstract Algebra; Quiz 2

0. Print your name: ______________________________

1(a). Suppose G is a group. We say that an element $a \in G$ *has a square root* if there is an element $b \in G$ such that $b^2 = a$. Find the error in the following argument:
"Suppose every element of a group G has a square root. And suppose G is not the trivial group. We will show that the group is infinite. Let $a_1 \in G$ with $a_1 \neq e$ the identity. Take a_2 to be a square root of a_1. Observe that since $a_2 a_2 = a_1$ it follows that $a_2 \neq a_1$ (for if $a_1 = a_2$ then we could cancel one of terms and then a_1 would the identity, contradicting our choice of a_1). Now take a_3 to be a square root of a_2. As before, $a_3 \neq a_2$. Continuing in this way, we have an infinite sequence of elements $a_1, a_2, a_3 \ldots \in G$. Thus G is infinite."

(b). Show that Z_3 is a counterexample to the result the above argument purports to prove.

2. Define the terms: "order of an element" and "order of a subgroup".

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Abstract Algebra; Quiz 3

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0. Print your name: ______________________________

1. Express (2 5 7)(5 1 4 3)(3 9 7 2 1) as a product of disjoint cycles.

2. Find the inverse of (1 4 7 8)(3 5)

3. What are all the possible orders of elements of S_6 ? Justify and explain your answer.

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Abstract Algebra; Quiz 4

0. Print your name: ____________________________

1. Is $(1\quad 5\quad 8)(4\quad 3\quad 7\quad 9)$ odd or even ?

2. Find the index of the subgroup of S_7 generated by $(1\quad 2\quad 3)(4\quad 5)$.

3. Find the cosets of the subgroup of S_3 generated by $(1\quad 2\quad 3)$

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Abstract Algebra; Quiz 5

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1. Find $\Big[S_6 : \Big\langle (1\,2\,3\,4)(5\,6) \Big\rangle\Big]$

2. Find, up to isomorphism, all abelian groups of order 360.

3. Prove that any subgroup of index two must be normal.

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Abstract Algebra; Quiz 6

(Note: Exam 2 on April 13)

0. Print your name: ______________________________

1. Suppose $f : G \to H$ is a homomorphism.
(a). Prove that $f(1_G) = 1_H$.

(b). Prove that $f(x^{-1}) = (f(x))^{-1}$ for all $x \in G$.

2. Prove that the composition of homomorphisms is a homomorphism.