Math 518 Spring 2002 Exam 2 NAME:

1. Solve the following system of differential equations:

$$\begin{array}{rcrcrcrcr} y_1'(t) &=& y_1(t) &-& y_2(t) & \qquad & y_1(0) &=& 5 \\ y_2'(t) &=& y_1(t) &+& 3y_2(t) & \qquad & y_2(0) &=& -7 \end{array}.$$

2. Find e^{tA} for $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 3 & 0 \\ 0 & 1 & 1 \end{bmatrix}$.

3. Let V be a finite dimensional vector space and $T \in \mathcal{L}(V)$. The characteristic polynomial of T is $p(x) = (x-a)^3(x-b)^2(x-c)^4$ and the minimal polynomial is $p(x) = (x-a)^2(x-b)(x-c)^3$. a, b, c are pairwise distinct scalars. Find the Jordan Canonical Form of T.

4. Let V be a finite dimensional vector space, $T \in \mathcal{L}(V)$, and λ a nonzero scalar such that $T^2 = \lambda T$. Show that $V = V_\lambda \oplus V_0$, where $V_\lambda = \{v \in V \mid T(v) = \lambda v\}$ and $V_0 = \{v \in V \mid T(v) = 0\}$.

5. Let V be a finite dimensional vector space and $T \in \mathcal{L}(V)$. The characteristic polynomial of T is $p(x) = 5x^7 - 3x^4 + x^2 - x - 5$. Find a polynomial $q(x)$ such that $q(T) = T^{-1}$.

6. Let $T \in \mathcal{L}(V)$, m a positive integer, $v \in V$ such that $T^{m-1}(v) \neq 0$ but $T^m(v) = 0$. Prove that the set $\{v, T(v), T^2(v), ..., T^{m-1}(v)\}$ is linearly independent.

7. Let $\mathbf{P}_n(\mathbf{C})$ denote the vector space of polynomials in x with coefficients in $\mathbf{C}$ and degree $\leq n$.
Define $S : \mathbf{P}_n(\mathbf{C}) \to \mathbf{P}_n(\mathbf{C})$ as $S(p(x)) = p(x+1) - p(x-1)$. Find the minimal polynomial of S. Explain.

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