GY403 STRUCTURAL GEOLOGY

KINEMATIC ANALYSIS

KINEMATICS

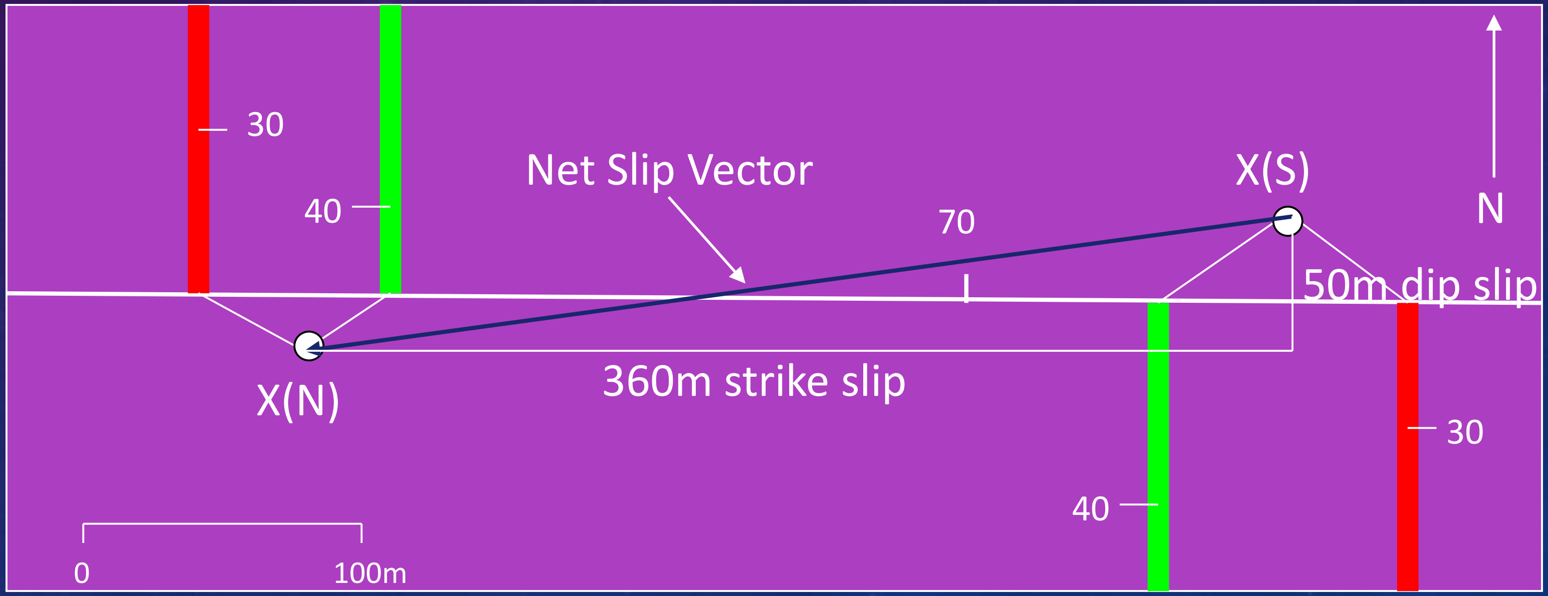
- Translation- described by a vector quantity
- Rotation- described by:
  - Axis of rotation point
  - Magnitude of rotation (degrees)
  - Sense of rotation (reference frame; clockwise or anticlockwise)
- Dilation- volume change
  - Loss of volume = negative dilation
  - Increase of volume = positive dilation
- Distortion- change in original shape

RIGID VS. NON-RIGID BODY DEFORMATION

- Rigid Body Deformation
  - Translation: fault slip
  - Rotation: rotational fault
- Non-rigid Body Deformation
  - Dilation: burial of sediment/rock
  - Distortion: ductile deformation (permanent shape change)

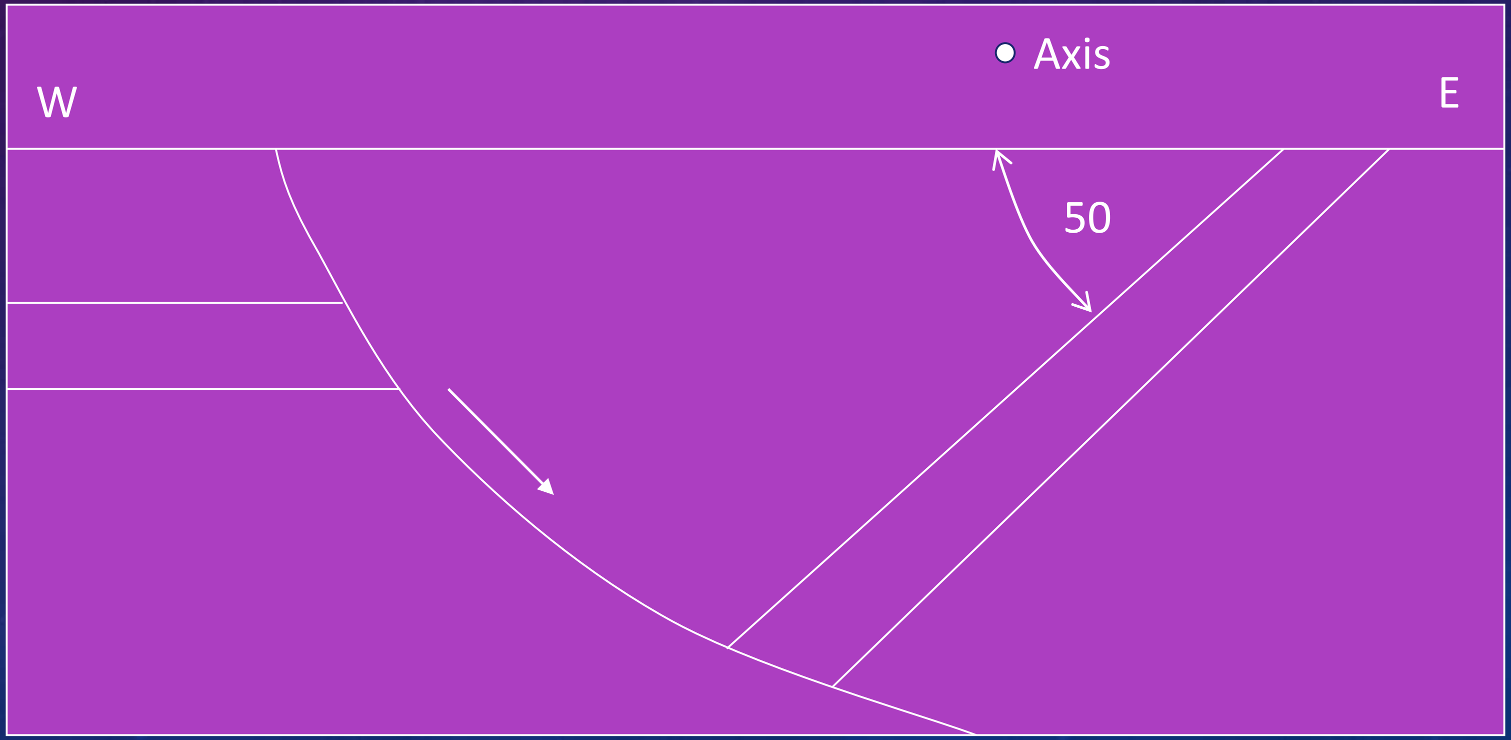
TRANSLATION EXAMPLES

- Slip along a planar fault
  - 360 meters left lateral slip
  - 50 meters normal dip slip
  - Classification: normal left-lateral slip fault



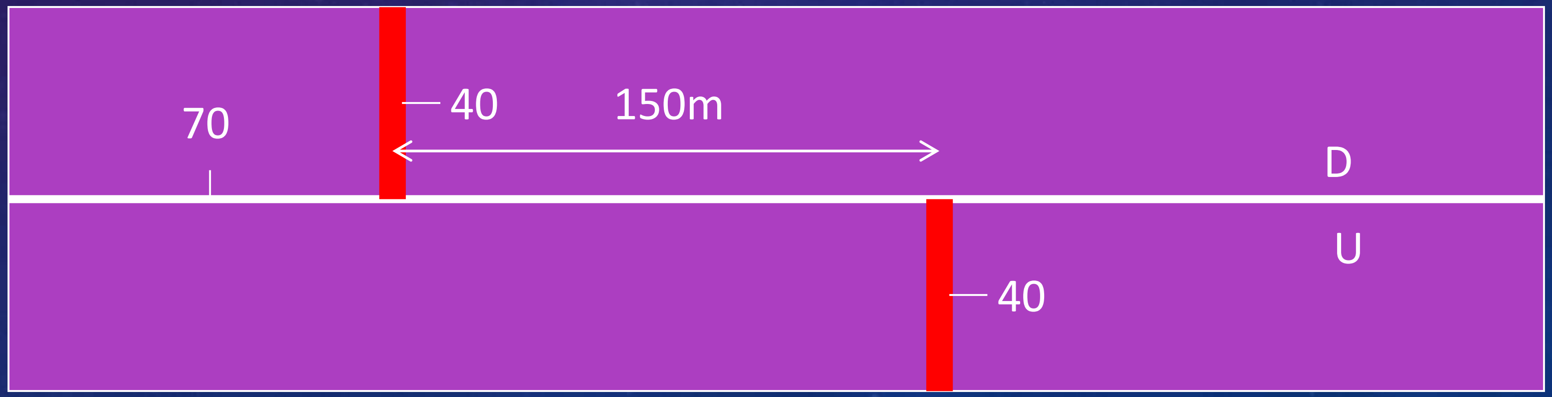
ROTATIONAL FAULT

- Fault slip is described by an axis of rotation
- Rotation is anticlockwise as viewed from the south fault block
- Amount of rotation is 50 degrees



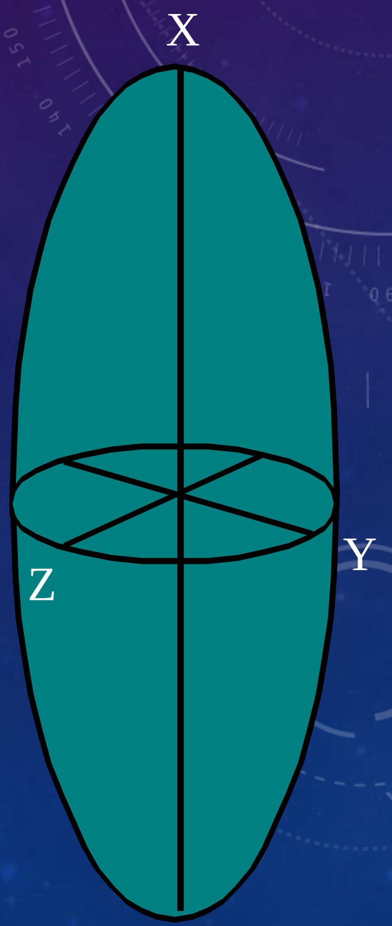
FAULT SEPARATION VS. SLIP

- Fault separation: the apparent slip as viewed on a planar outcrop.
- Fault slip: must be measured with net slip vector using a linear feature offset by the fault.



STRAIN ELLIPSOID

- A three-dimensional ellipsoid that describes the magnitude of dilational and distortional strain.
- Assume a perfect sphere before deformation.
- Three mutually perpendicular axes X, Y, and Z.
- X is maximum stretch (S_X) and Z is minimum stretch (S_Z).
- There are unique directions corresponding to values of S_X and S_Z, but an infinite number of directions corresponding to Sy.



STRAIN: THE RESULTS OF DEFORMATION FROM DISTORTION AND DILATION

- Heterogeneous strain: strain ellipsoid varies from point-to-point in deformed body
- Homogenous strain: strain ellipsoid is equivalent from point-to-point in deformed body
- Although heterogenous strain is the rule in real rocks, often portions of a deformed body behave as homogenous with respect to strain

GENERAL STRAIN EQUATIONS: EXTENSION (E), STRETCH (S), AND QUADRATIC ELONGATION (Λ)

These equations measure linear strain :

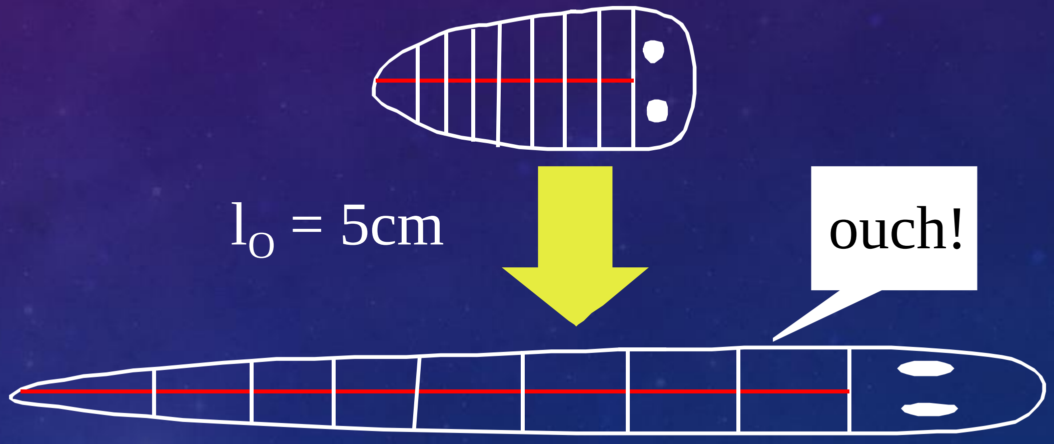
l_O = original length

l_F = final length

$$e = \frac{l_F - l_O}{l_O}$$

$$S = \frac{l_F}{l_O}$$

$$\lambda = \left[\frac{l_F}{l_O}\right]^2$$



l_O = 5cm

l_F = 12cm

$$S = \frac{l_F}{l_O} = \frac{12\text{cm}}{5\text{cm}} = 2.4$$

$$e = (S-1) = 2.4 - 1 = 1.4$$

$$\lambda = S^2 = (2.4)^2 = 5.76$$

ROTATIONAL STRAIN EQUATIONS: QUANTIFYING ANGULAR SHEAR (Ψ) AND SHEAR STRAIN (Γ)

- Note the convention that positive theta is measured clockwise form X, negative theta counterclockwise from X.
- The convention for alpha is:
  - "+" for clockwise rotation from undeformed to deformed state (i.e. L' to L)
  - "-" for counterclockwise rotation from undeformed to deformed state.

θ = angle between reference line (L) and maximum stretch (X) (clockwise=+; anticlockwise=-)

ψ_L (perpendicular to L relative to M) = -40

$\gamma_L = \tan(\psi_L) = \tan(-40) = -0.839$

$\alpha_L = \theta_d - \theta = (-25) - (-35) = +10$
angle of internal rotation

MOHR CIRCLE FOR STRAIN:

- General equations as a function of λ_X, λ_Z, and θ_d .

$$\lambda' = \frac{1}{\lambda}$$

λ_X = quadratic elongation parallel to X axis of finite strain ellipse

λ_Z = quadratic elongation parallel to Z axis of finite strain ellipse

$$\lambda' = \frac{\lambda'_Z + \lambda'_X}{2} - \frac{\lambda'_Z - \lambda'_X}{2} \cos(2\theta_d)$$

$$\frac{\gamma}{\lambda} = \frac{\lambda'_Z - \lambda'_X}{2} \sin(2\theta_d)$$

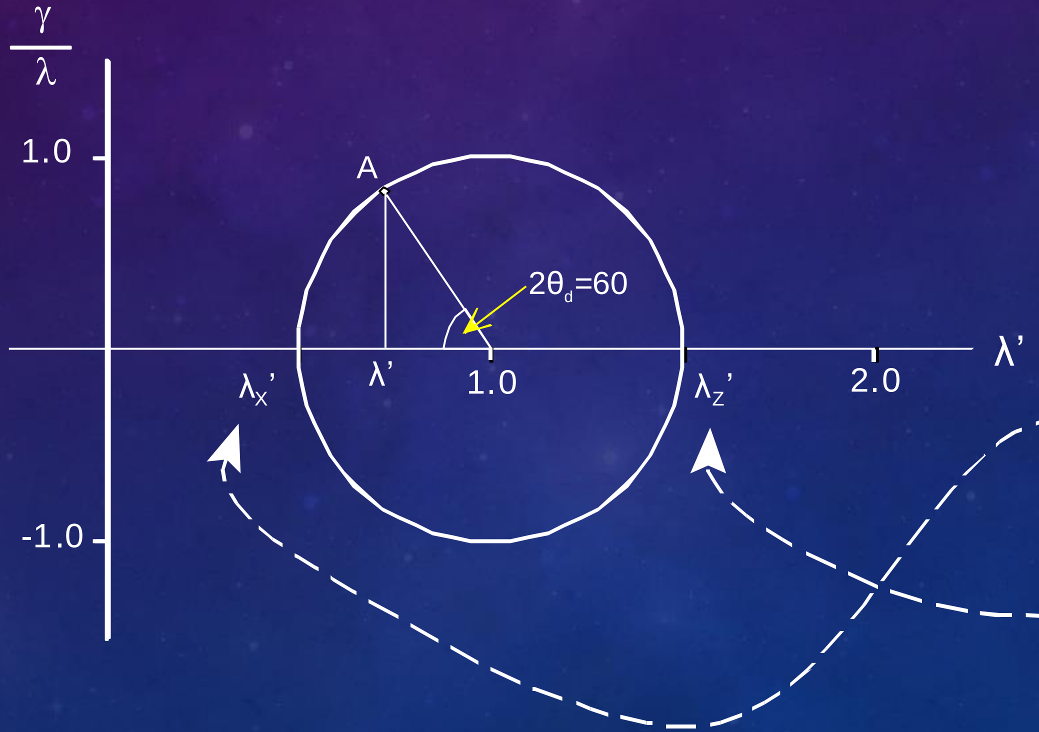
$$\tan \theta_d = \tan \theta \, \frac{S_Z}{S_X}$$

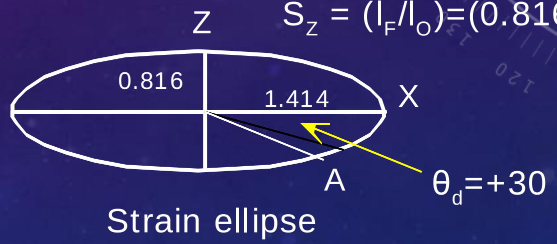
$$\alpha = \theta_d - \theta$$

(internal rotation)

MOHR CIRCLE FOR STRAIN: GEOMETRIC RELATIONS BETWEEN THE FINITE STRAIN ELLIPSE AND THE MOHR CIRCLE FOR STRAIN

- Mohr circle for strain as a function of Lambda prime vs. Gamma/(shear strain).
- Note that the 360 degrees of the Mohr circle represents 180 degrees in reality (+X to –X on the finite strain ellipse).





$S_X = (l_F/l_O) = (1.414/1.0) = 1.414$

$S_Z = (l_F/l_O) = (0.816/1.0) = 0.816$

$\lambda_X = (S_x)^2 = (1.414)^2 = 2.0$

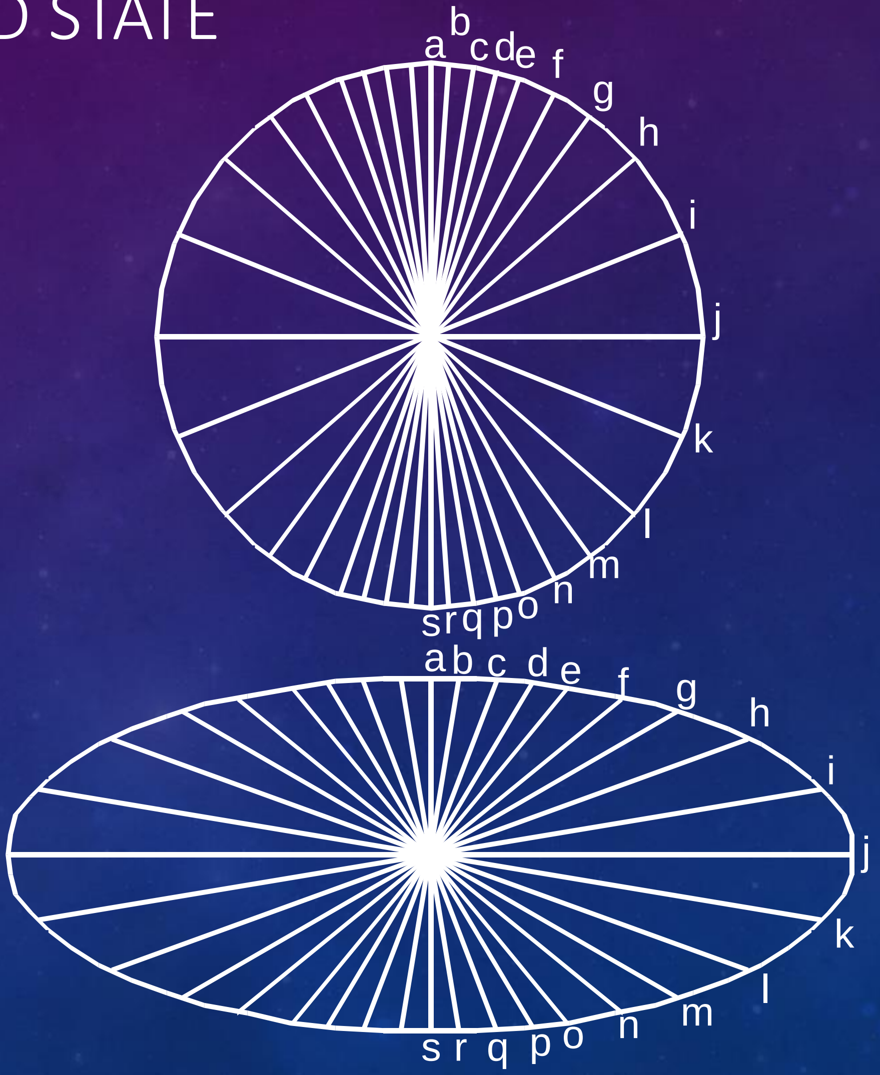
$\lambda_X' = 1/\lambda = 1/2.0 = 0.5$

$\lambda_Z = (S_z)^2 = (0.816)^2 = 0.666$

$\lambda_Z' = 1/\lambda = 1/0.666 = 1.50$

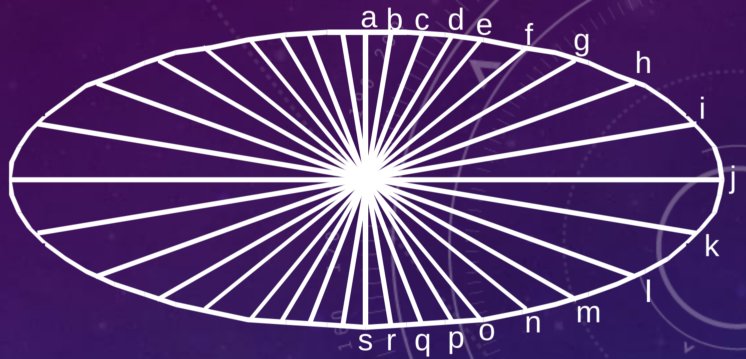
MOHR CIRCLE FOR STRAIN: REFERENCE LINES IN THE UNDEFORMED AND DEFORMED STATE

- The starting circle represents the initial pre-deformation geometry.
- After deformation the finite strain ellipse represents the distortion and dilation components of strain.



$S_X=1.936$
$S_Z=0.707$

MOHR CIRCLE STRAIN RELATIONSHIPS



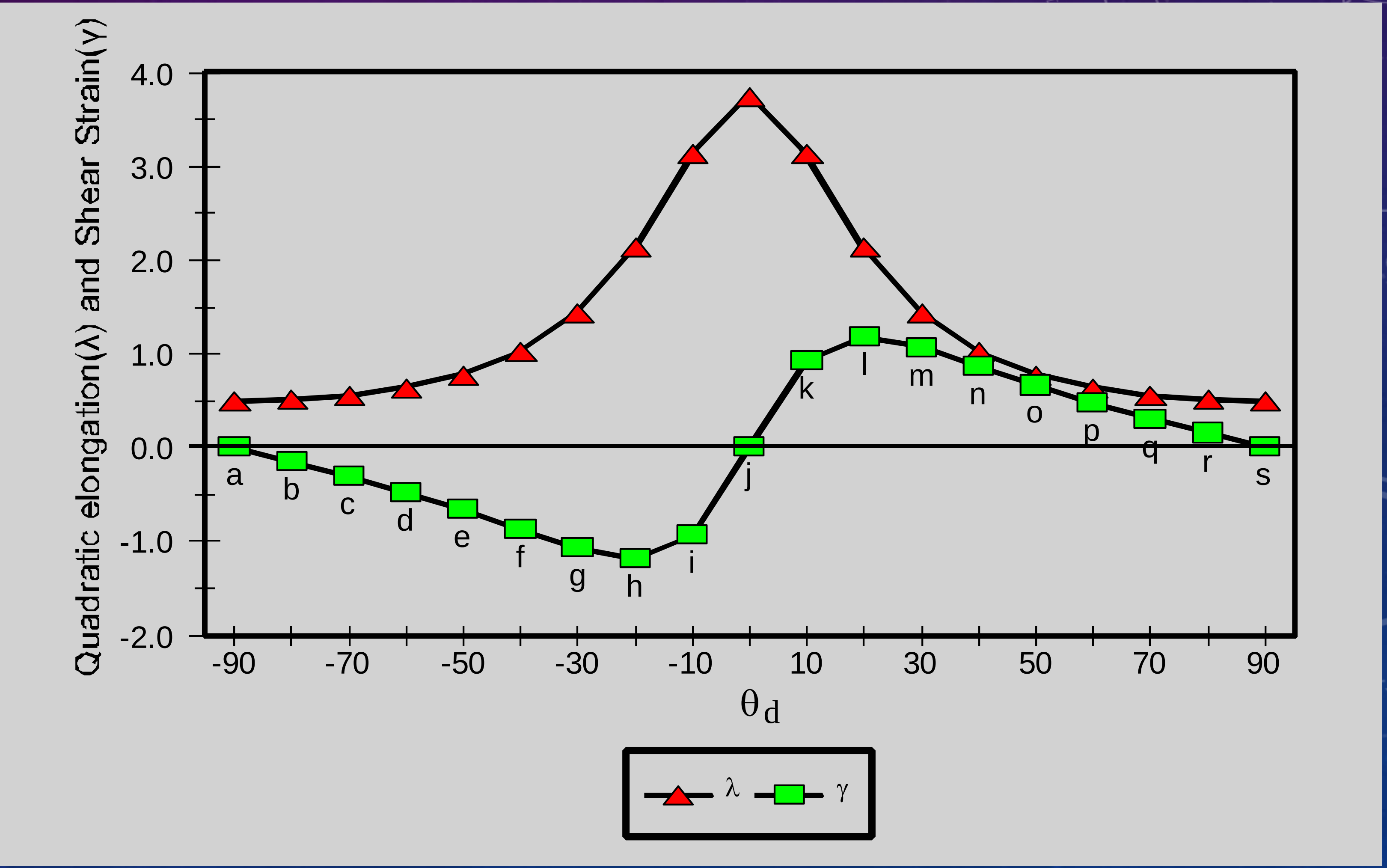
- Values of quadratic elongation (λ), shear strain (γ), original θ angle, angular shear (ψ), and angle of internal rotation (α) as a function of θ_d

Line	θ_d	λ	γ	θ	ψ
a	-90	0.500	-0.000	-90.0	-0.0
b	-80	0.513	-0.152	-86.3	-8.7
c	-70	0.556	-0.310	-82.4	-17.2
d	-60	0.638	-0.479	-78.1	-25.6
e	-50	0.779	-0.665	-73.0	-33.6
f	-40	1.017	-0.868	-66.5	-41.0
g	-30	1.428	-1.072	-57.7	-47.0
h	-20	2.129	-1.187	-44.9	-49.9
i	-10	3.134	-0.929	-25.8	-42.9
j	0	3.748	0.000	0.0	0.0
k	10	3.134	0.929	25.8	42.9
l	20	2.129	1.187	44.9	49.9
m	30	1.428	1.072	57.7	47.0
n	40	1.017	0.868	66.5	41.0
o	50	0.779	0.665	73.0	33.6
p	60	0.638	0.479	78.1	25.6
q	70	0.556	0.310	82.4	17.2
r	80	0.513	0.152	86.3	8.7
s	90	0.500	0.000	90.0	0.0

$S_X=1.936$
$S_Z=0.707$

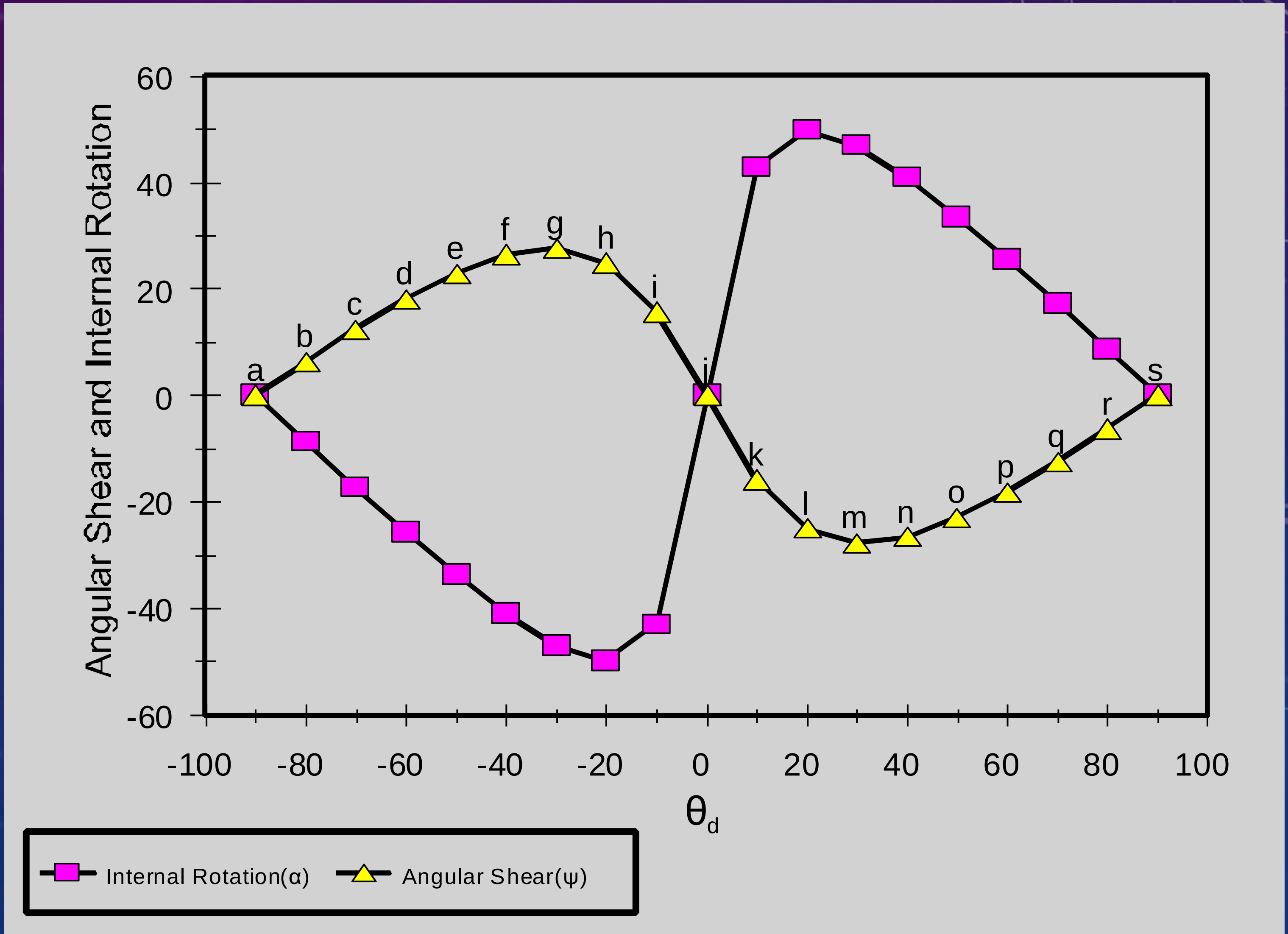
STRAIN ELLIPSE GENERAL EQUATION

- Values for quadratic elongation (λ) and shear strain (γ) as a function of θ_d



STRAIN ELLIPSE GENERAL EQUATION

- Values for angular shear (ψ) and internal rotation (α) as a function of θ_d



EXAMPLE STRAIN PROBLEM

- Given a finite strain ellipse of S_X=1.936 and S_Z=0.707, find for direction θ_d=-20̊ values of S, λ, γ, ψ, and α

$\lambda_X=(1.936)^2 = 3.750;\ \lambda_Z = (0.707)^2 = 0.500;\ \lambda'_X=0.267;\ \lambda'_Z=2.0$

$$\lambda' = \frac{2.0+0.267}{2} - \frac{2.0 - 0.267}{2}\ \text{Cos}(-40) = 1.133-(0.866)(0.766) = 0.470$$

$\lambda = 1/\lambda' = 1/0.470 = 2.128 \quad \therefore\ S = (2.128)^{0.5} = 1.459$

$$\gamma = \frac{2.0-0.267}{2}\ \text{Sin}(-40)\ \lambda = (0.866)(-0.643)(2.128) = -1.185$$

$\psi = \tan^{-1}(\gamma) = \tan^{-1}(-1.185) = -49.8°$

$$\text{Tan}(\theta_d) = \tan(\theta)\ \frac{S_Z}{S_X} \quad \therefore \quad \text{Tan}(\theta) = \tan(\theta_d)\ \frac{S_X}{S_Z} \quad \therefore \quad \theta = -44.9°$$

$\alpha = \theta_d - \theta = (-20) - (-44.9) = +24.9°$

APPLICATION OF PLANE STRAIN

- Deformed ooids from the study of Cloos (1947).

Assuming plane strain: no dilational component to strain, therefore, constant volume applies:

$V_S = 4/3\pi r^3$ where r is the radius of the sphere

$V_e = 4/3\pi abc$ where (a,b,c) are the ½ axial legths of the ellipsoid

$V_s = V_e$

$4/3\pi r^3 = 4/3\pi abc$

Because of plane strain $r = b \therefore$

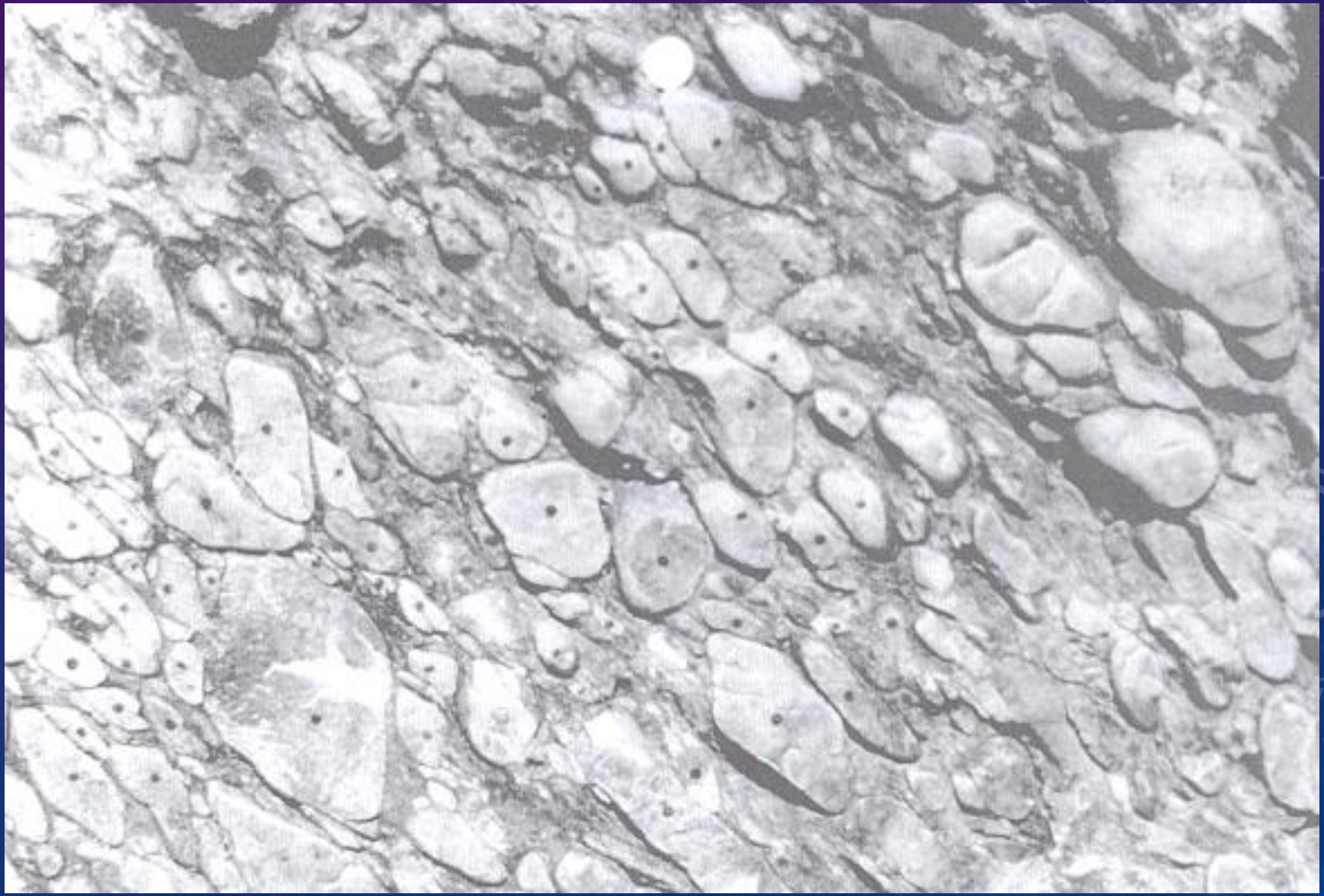
$r^2 = ac$

$r = (ac)^{0.5}$

Example: a=4.2mm; c=2.5mm; $r=(4.2*2.5)^{0.5} = 3.3 \therefore S_x = 4.2/3.3 = 1.27$

APPLICATION TO DEFORMED STRAIN MARKERS

- Markers may be original spheres or ellipsoids
- Pebbles, sand grains, reduction spots, ooids, fossils, etc.
- Assume homogenous strain domain



MEASURING LENGTH/WIDTH RATIOS (R_F)

- Measure major and minor axis of each strain ellipse.
- Rf = (Major/minor) (yields a unitless ratio).
- ϕ = Angle from reference direction (usually foliation or cleavage), positive angles are clockwise, negative counterclockwise.

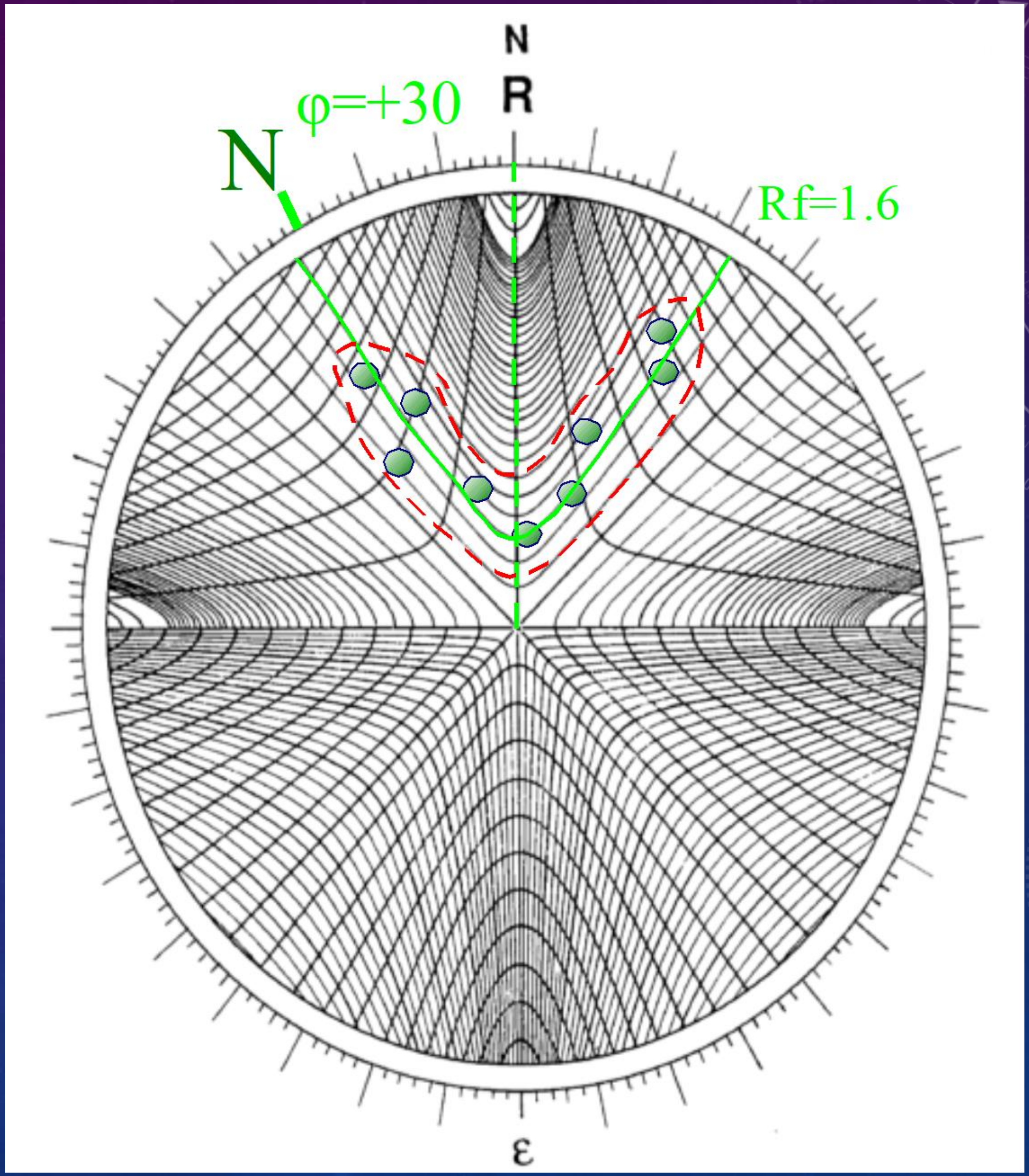
SPREADSHEET SETUP FOR RF/Φ ANALYSIS

- Note: ϕ is measured relative to a chosen reference direction such as foliation

Ellipse	Length	Width	Rf	ϕ
1	0.3066	0.1600	1.916	32.8
2	0.0969	0.0704	1.376	51.9
3	0.1221	0.0729	1.675	61.8
4	0.0660	0.0389	1.697	54.6
5	0.1735	0.1392	1.246	22.7
6	0.0825	0.0539	1.531	76.2
7	0.1770	0.1275	1.388	67.5
8	0.0736	0.0347	2.121	37.6
9	0.0937	0.0797	1.176	-0.3
10	0.1184	0.0457	2.591	10.6

HYPERBOLIC NET

- Used to plot strain markers that were originally ellipsoidal
- Statistically the R_f ratios will tend to fall along one of the hyperbolic curves

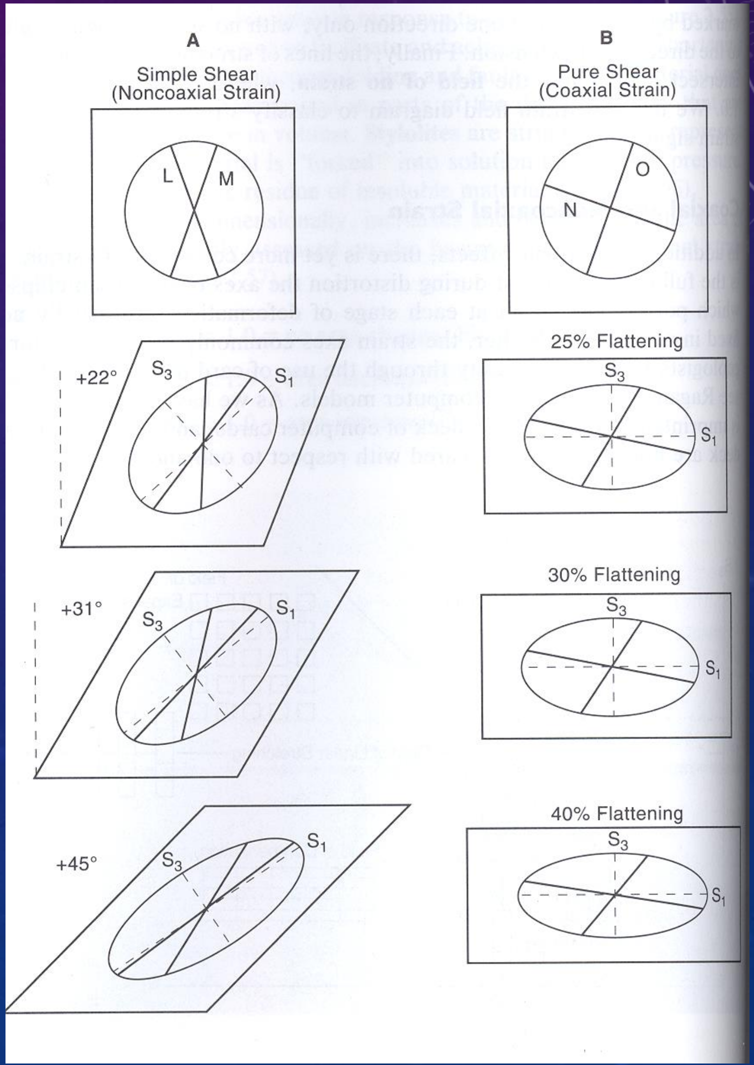


PURE SHEAR VS. SIMPLE SHEAR

- Pure Shear: also known as coaxial strain and/or flattening strain.
    - Principle axes of finite strain ellipse remain at same orientation through progressive strain
- Simple Shear: also know as non-coaxial strain.
    - Principle axes of finite strain ellipse rotate through each stage of progressive strain.

SIMPLE SHEAR VS. PURE SHEAR

- Simple shear mechanism in contrast to pure shear.
- Pure shear is also known as coaxial strain because the principle XYZ axes do not change their orientation through increments of strain



SUMMARY FOR EXAMS

- Know how to interpret the Rf vs. ϕ diagram.
- Know the 4 components of kinematic analysis, including rigid vs. non-rigid body deformation, and geological examples.
- Know the definition of homogenous vs. non-homogenous strain and geological examples.
- Know the general equations for the Mohr circle for strain, and how to calculate S, e, λ, γ, Ψ, and α from the axial lengths of the finite strain ellipse.
- Be familiar with Ernst Cloos (1947) study of deformed ooids and the relationship to plane strain.
- Be familiar with the concept of pure shear vs. simple shear .
- Know the definitions of fault separation versus fault slip; be able to classify a fault based on the net slip vector.