GY403 STRUCTURAL GEOLOGY

KINEMATIC ANALYSIS
KINEMATICS

• Translation- described by a vector quantity
• Rotation- described by:
  • Axis of rotation point
  • Magnitude of rotation (degrees)
  • Sense of rotation (reference frame; clockwise or anticlockwise)
• Dilation- volume change
  • Loss of volume = negative dilation
  • Increase of volume = positive dilation
• Distortion- change in original shape
RIGID VS. NON-RIGID BODY DEFORMATION

- Rigid Body Deformation
  - Translation: fault slip
  - Rotation: rotational fault
- Non-rigid Body Deformation
  - Dilation: burial of sediment/rock
  - Distortion: ductile deformation (permanent shape change)
TRANSLATION EXAMPLES

• Slip along a planar fault
  • 360 meters left lateral slip
  • 50 meters normal dip slip
  • Classification: normal left-lateral slip fault
ROTATIONAL FAULT

- Fault slip is described by an axis of rotation
- Rotation is anticlockwise as viewed from the south fault block
- Amount of rotation is 50 degrees
FAULT SEPARATION VS. SLIP

- Fault separation: the apparent slip as viewed on a planar outcrop.
- Fault slip: must be measured with net slip vector using a linear feature offset by the fault.
STRAIN ELLIPSOID

• A three-dimensional ellipsoid that describes the magnitude of dilational and distortional strain.
• Assume a perfect sphere before deformation.
• Three mutually perpendicular axes X, Y, and Z.
• X is maximum stretch ($S_X$) and Z is minimum stretch ($S_Z$).
• There are unique directions corresponding to values of $S_X$ and $S_Z$, but an infinite number of directions corresponding to $S_Y$. 
STRAIN: THE RESULTS OF DEFORMATION FROM DISTORTION AND DILATION

• Heterogeneous strain: strain ellipsoid varies from point-to-point in deformed body

• Homogenous strain: strain ellipsoid is equivalent from point-to-point in deformed body

• Although heterogenous strain is the rule in real rocks, often portions of a deformed body behave as homogenous with respect to strain
GENERAL STRAIN EQUATIONS: EXTENSION (E), STRETCH (S), AND QUADRATIC ELONGATION (Λ)

These equations measure linear strain:

\[ l_F = \text{final length} \]
\[ l_O = \text{original length} \]

\[ e = \frac{l_F - l_O}{l_O} \]
\[ S = \frac{l_F}{l_O} \]
\[ \lambda = \left( \frac{l_F}{l_O} \right)^2 \]

- \( l_O = 5\text{cm} \)
- \( l_F = 12\text{cm} \)

\[ S = \frac{l_F}{l_O} = \frac{12\text{cm}}{5\text{cm}} = 2.4 \]

\[ e = (S-1) = 2.4 - 1 = 1.4 \]
\[ \lambda = S^2 = (2.4)^2 = 5.76 \]
ROTATIONAL STRAIN EQUATIONS: QUANTIFYING ANGULAR SHEAR ($\Psi$) AND SHEAR STRAIN ($\Gamma$)

- Note the convention that positive theta is measured clockwise from X, negative theta counterclockwise from X.
- The convention for alpha is:
  - “+” for clockwise rotation from undeformed to deformed state (i.e. L’ to L)
  - “-” for counterclockwise rotation from undeformed to deformed state.

$\theta = \text{angle between reference line (L) and maximum stretch (X)}$ (clockwise=+; anticlockwise=-)

$\psi_L$ (perpendicular to L relative to M) = -40

$\gamma_L = \tan(\psi_L) = \tan(-40) = -0.839$

$\alpha_L = \theta_d - \theta = (-25) - (-35) = +10$

Angle of internal rotation
MOHR CIRCLE FOR STRAIN:

• General equations as a function of $\lambda_x$, $\lambda_z$, and $\theta_d$.

\[
\lambda' = \frac{1}{\lambda}
\]

$\lambda_x = \text{quadratic elongation parallel to X axis of finite strain ellipse}$

$\lambda_z = \text{quadratic elongation parallel to Z axis of finite strain ellipse}$

\[
\lambda' = \frac{\lambda'_z + \lambda'_x - \lambda'_z - \lambda'_x \cos(2\theta_d)}{2}
\]

\[
\gamma = \frac{\lambda'_z - \lambda'_x \sin(2\theta_d)}{2}
\]

\[
\frac{\lambda'}{\lambda} = \frac{S_z}{S_x}
\]

$\alpha = \theta_d - \theta$

(internal rotation)
MOHR CIRCLE FOR STRAIN: GEOMETRIC RELATIONS BETWEEN THE FINITE STRAIN ELLIPSE AND THE MOHR CIRCLE FOR STRAIN

- Mohr circle for strain as a function of Lambda prime vs. Gamma/(shear strain).
- Note that the 360 degrees of the Mohr circle represents 180 degrees in reality (+X to –X on the finite strain ellipse).

\[
\begin{align*}
\lambda' &= \frac{S_x}{l_F} = \frac{1.414}{1.0} = 1.414 \\
\lambda'' &= \frac{S_z}{l_F} = \frac{0.816}{1.0} = 0.816
\end{align*}
\]

\[
\begin{align*}
\lambda_x &= (S_x)^2 = (1.414)^2 = 2.0 \\
\lambda_x' &= \frac{1}{\lambda} = \frac{1}{2.0} = 0.5 \\
\lambda_z &= (S_z)^2 = (0.816)^2 = 0.666 \\
\lambda_z' &= \frac{1}{\lambda} = \frac{1}{0.666} = 1.50
\end{align*}
\]
MOHR CIRCLE FOR STRAIN: REFERENCE LINES IN THE UNDEFORMED AND DEFORMED STATE

- The starting circle represents the initial pre-deformation geometry.
- After deformation the finite strain ellipse represents the distortion and dilation components of strain.

\[ S_x = 1.936 \]
\[ S_z = 0.707 \]
## MOHR CIRCLE STRAIN RELATIONSHIPS

- Values of quadratic elongation ($\lambda$), shear strain ($\gamma$), original $\theta$ angle, angular shear ($\psi$), and angle of internal rotation ($\alpha$) as a function of $\theta_d$.

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<th>Line</th>
<th>$\theta_d$</th>
<th>$\lambda$</th>
<th>$\gamma$</th>
<th>$\theta$</th>
<th>$\psi$</th>
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<td>0.500</td>
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$S_X=1.936$

$S_Z=0.707$
Values for quadratic elongation ($\lambda$) and shear strain ($\gamma$) as a function of $\theta_d$
STRAIN ELLIPSE GENERAL EQUATION

- Values for angular shear ($\psi$) and internal rotation ($\alpha$) as a function of $\theta_d$
EXAMPLE STRAIN PROBLEM

- Given a finite strain ellipse of \( S_X = 1.936 \) and \( S_Z = 0.707 \), find for direction \( \theta_d = -20^\circ \) values of \( S, \lambda, \gamma, \psi, \) and \( \alpha \)

\begin{align*}
\lambda_X &= (1.936)^2 = 3.750; \\
\lambda_Z &= (0.707)^2 = 0.500; \\
\lambda'_X &= 0.267; \\
\lambda'_Z &= 2.0
\end{align*}

\begin{align*}
\lambda' &= \frac{2.0 + 0.267 - 2.0 - 0.267}{2} \\
\lambda &= 1/\lambda' = 1/0.470 = 2.128 \quad \therefore S = (2.128)^{0.5} = 1.459
\end{align*}

\begin{align*}
\gamma &= \frac{2.0 - 0.267}{2} \\
\sin(-40) \lambda &= (0.866)(-0.643)(2.128) = -1.185 \\
\psi &= \tan^{-1}(\gamma) = \tan^{-1}(-1.185) = -49.8^\circ
\end{align*}

\begin{align*}
\tan(\theta_d) &= \tan(\theta) \frac{S_Z}{S_X} \quad \therefore \quad \tan(\theta) = \tan(\theta_d) \frac{S_X}{S_Z} \quad \therefore \quad \theta = -44.9^\circ
\end{align*}

\begin{align*}
\alpha &= \theta_d - \theta = (-20) - (-44.9) = +24.9^\circ
\end{align*}
Deformed ooids from the study of Cloos (1947).

Assuming plane strain: no dilational component to strain, therefore, constant volume applies:

\[ V_s = \frac{4}{3} \pi r^3 \] where \( r \) is the radius of the sphere

\[ V_e = \frac{4}{3} \pi abc \] where \( a, b, c \) are the \( \frac{1}{2} \) axial lengths of the ellipsoid

\[ V_s = V_e \]

\[ 4/3 \pi r^3 = 4/3 \pi abc \]

Because of plane strain \( r = b \therefore \)

\[ r^2 = ac \]

\[ r = (ac)^{0.5} \]

Example: \( a = 4.2 \text{mm}; c = 2.5 \text{mm}; r = (4.2 \times 2.5)^{0.5} = 3.3 \therefore S_x = 4.2/3.3 = 1.27 \]
APPLICATION TO DEFORMED STRAIN MARKERS

• Markers may be original spheres or ellipsoids
• Pebbles, sand grains, reduction spots, ooids, fossils, etc.
• Assume homogenous strain domain
MEASURING LENGTH/WIDTH RATIOS ($R_F$)

- Measure major and minor axis of each strain ellipse.
- $R_f = \frac{\text{Major}}{\text{minor}}$ (yields a unitless ratio).
- $\phi = \text{Angle from reference direction (usually foliation or cleavage), positive angles are clockwise, negative counterclockwise.}$
SPREADSHEET SETUP FOR RF/Φ ANALYSIS

- Note: $\phi$ is measured relative to a chosen reference direction such as foliation.

<table>
<thead>
<tr>
<th>Ellipse</th>
<th>Length</th>
<th>Width</th>
<th>Rf</th>
<th>$\phi$</th>
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<td>0.1600</td>
<td>1.916</td>
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</table>
HYPERBOLIC NET

• Used to plot strain markers that were originally ellipsoidal

• Statistically the $R_f$ ratios will tend to fall along one of the hyperbolic curves
PURE SHEAR VS. SIMPLE SHEAR

• Pure Shear: also known as coaxial strain and/or flattening strain.
  • Principle axes of finite strain ellipse remain at same orientation through progressive strain
• Simple Shear: also know as non-coaxial strain.
  • Principle axes of finite strain ellipse rotate through each stage of progressive strain.
SIMPLE SHEAR VS. PURE SHEAR

• Simple shear mechanism in contrast to pure shear.
• Pure shear is also known as coaxial strain because the principle XYZ axes do not change their orientation through increments of strain
SUMMARY FOR EXAMS

• Know how to interpret the Rf vs. φ diagram.
• Know the 4 components of kinematic analysis, including rigid vs. non-rigid body deformation, and geological examples.
• Know the definition of homogenous vs. non-homogenous strain and geological examples.
• Know the general equations for the Mohr circle for strain, and how to calculate S, e, λ, γ, Ψ, and α from the axial lengths of the finite strain ellipse.
• Be familiar with Ernst Cloos (1947) study of deformed ooids and the relationship to plane strain.
• Be familiar with the concept of pure shear vs. simple shear.
• Know the definitions of fault separation versus fault slip; be able to classify a fault based on the net slip vector.